Системы квантовых точек

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Идеи и методы физики конденсированного состояния,XIV школа-конференция "Проблемы физики твердого тела и высоких давлений", 11 – 20 сентября, 2015. Буревестник, Сочи

Subjects

- Single quantum dot – gigantic artificial atoms

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- Coupled quantum dots – QD molecules

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- QD arrays can form artificial crystals and quasicrystals with tunable properties.

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- Single quantum dot – gigantic artificial atoms
- Coupled quantum dots – QD molecules
- QD arrays can form artificial crystals and quasicrystals with tunable properties.
- External field control
- Spontaneous coherence effects

Applications of QD systems

- Optoelectronics
  - Lasers on quantum wells and quantum dots
  - Optical modulators
  - Photodetector on quantum wells
  - Solar Cells
  - Waveguides
- Nanoelectronics
  - Qubits
  - Gates
  - Quantum memory (with hierarchic access)
  - Quantum neural nets

Quantum dots (QD) are "artificial atoms the quantum analog of the Thomson atom. They form the complexes - the "molecules"of QDs

The tunable parameters:

- the number of charge carriers and
- their localization.

Exciton states participate in

- kinetic phenomena,
- the luminescence,
- light absorption,
- coherent radiation generation,
- chemisorption
- heat conductivity etc.

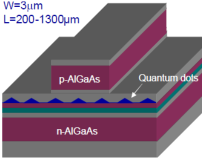
Comparison between QDs and atoms

Parameter	Atoms	Quantum dots
Level spacing	1 eV	0.1 meV
Ionization energy	10 eV	0.1 meV
Typical magnetic field	10^4 T	1-10 T

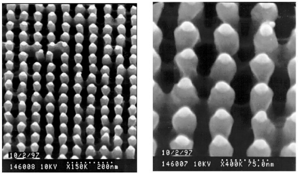
Methods of QD production

The quantum dot and quantum well structures can produced by different experimental technique:

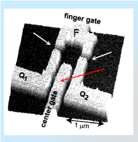
- molecular beam epitaxy,



- lithography,

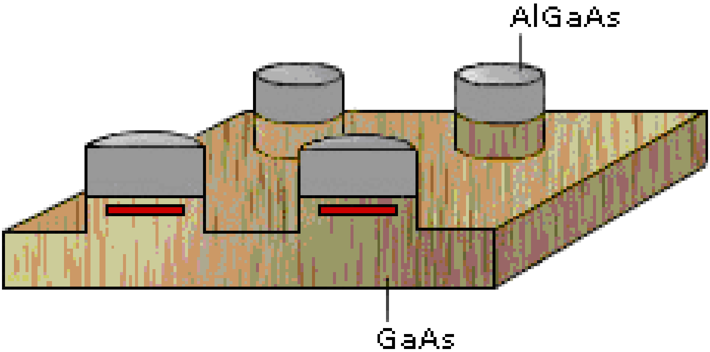


- selective etching



- gate voltage application.

Etching process for manufacturing quantum dot array from the dopped $AlGaAs/GaAs$ heteropair



”Natural” QDs

Besides artificially produced quantum dots the so-called ”natural” quantum dots may occur due to random fluctuations.

Effects of external fields

$$\hat{H} = \sum_i \left[\frac{1}{2m^*}\left(-\imath\hbar\nabla_i + \frac{e}{c}\boldsymbol{A}\right)^2 + \alpha r_i^2\right] + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{\epsilon r_{ij}}$$

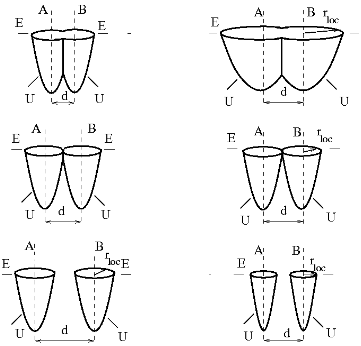
We use symmetric gauge of vector potential $\boldsymbol{A} = \frac{1}{2}[\boldsymbol{B}\boldsymbol{r}]$
The influence of transversal magnetic field on the quantum dot leads to the substitution of the steepness parameter α by effective steepness of confining potential in magnetic field

$$\beta = \left[(\omega_C/4)^2 + \alpha\right]^{1/2}$$

Arbitrary units for energy, length, cyclotronic frequency and magnetic field

	Electronic systems	Excitonic systems
Length	$a_0 = \frac{\hbar^2 \varepsilon}{2 m_e^* e^2}$	$a_0 = \frac{\hbar^2 \varepsilon}{2 m^* e^2}$
Energy	$E_0 = \frac{2 m_e^* e^4}{\hbar^2 \varepsilon^2}$	$E_0 = \frac{2 m^* e^4}{\hbar^2 \varepsilon^2}$
Steepness of the potential	$\alpha_0 = \frac{E_0}{a_0^2}$	$\alpha_0 = \frac{E_0}{a_0^2}$
Magnetic field	$B_0 = \frac{(2 m_e^*)^2 e^3 c}{\hbar^3 \varepsilon^2}$	$B_0 = \frac{2 (m^*)^2 e^3 c}{\hbar^3 \varepsilon^2}$
Cyclotronic frequency	$\omega_{c_0} = \frac{4 m_e^* e^4}{\varepsilon^2 \hbar^3}$	$\omega_{c_0} = \frac{2 m^* e^4}{\varepsilon^2 \hbar^3}$
Larmore frequency		$\omega_{L_0} = \frac{m^* e^4}{\varepsilon^2 \hbar^3} = \frac{\omega_{c_0}}{2}$

г ε – dielectric permeability; $m_{e,h}^*$ – effective mass of electron (e) and hole (h); $m^* = \frac{m_e^* m_h^*}{m_e^* + m_h^*}$ – reduced exciton mass



with increase of inter-dot distance

with increase of effective steepness in magnetic field

$d < r_{loc}$, MO, V

$d \sim r_{loc}$ V

$d > kr_{loc}$ $(k \sim 3)$ HL, V

HL - Heitler-London; MO - Molecular orbitals; V - variation

Spin-rearrangement in quantum dots

Spin states as well as Coulomb interaction between electrons in coupled QD can be used for quantum computation.
Spin states have substantially longer decoherence time.
Mechanism of temporal fluctuations in B_n , which can occur due to nuclear dipole-dipole interactions, lead to irreversible spin dephasing and decoherence of electron spins. Such processes are referred to as spectral diffusion, since the electron Zeeman levels split by B_n undergo random shifts.
Estimates of hyperfine-interaction spin dephasing in GaAs quantum dots were given by de Sousa and Das Sarma (2005). That mechanism of temporal fluctuations in B_n,should dominate spin decoherence in GaAs quantum dots of radius smaller than 100 nm. For instance, in a 50-nm-wide quantum dot, the estimated spin decoherence time τ_{sc} is $\approx 50\mu s$, large enough for quantum computing applications.
This is important for quantum calculations and more generally for spintronics.

With increase of the transverse magnetic field, the triplet state becomes the ground state of the QD system.

There are two contributions connected with the magnetic field:

1. rise of the effective steepness of the confining potential leading to diminishing of the interdot coupling;
2. interaction of spins with the magnetic field.

Thus it is possible to control the spin state of a QD molecule by normal and parallel magnetic fields.

The possibility of controlling the ground state of a coupled QD system can be used for quantum calculations.

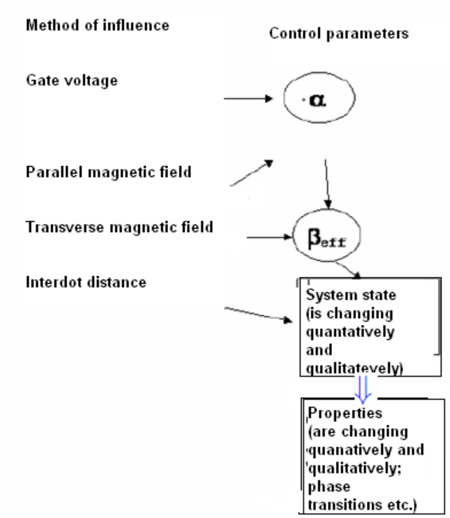
- One qubit can be determined by the spin of excess electrons in QDs.
- A two-qubit gate can be created using two coupled QDs.

The tunnel barrier between two adjacent QDs can be controlled by the gate voltage or by the external magnetic field.

Kaputkina and Lozovik, J. Phys.: Cond. Mat. 18(2006)S2169

Means of control QDs and QD systems

- Quantitative change of the state of the system
- Qualitative change of the state of the system
  - Strong electron correlation
  - Merging and separation of neighbouring QD
  - Spin state: singlet – triplet transition
  - Tunneling
  - etc

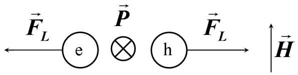


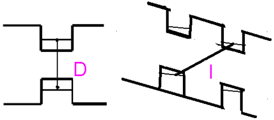
Single and coupled quantum wells and quantum dots in microcavity

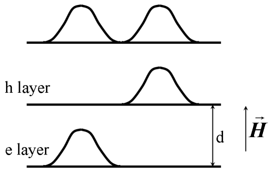
External magnetic field influence on

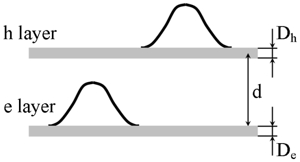
- Direct and indirect excitons in coupled quantum wells
  - quasi2D excitons
  - 3D excitons
- Direct and indirect excitons in coupled quantum dots
- Exciton polaritons in CQWs and in CQDs in microcavities

Spatially indirect exciton in coupled quantum wells









$$\psi(\vec{r}) = \Phi(\vec{r} - \vec{\rho}_0) \exp\left(\frac{\imath\gamma\vec{r}\cdot\vec{P}}{2\hbar}\right); \vec{\rho}_0 = \frac{c}{eH^2}[\vec{H}, \vec{P}]; \vec{\rho} = \vec{r} - \vec{\rho}_0$$

Relative motion of the electron and hole

$$\left[\Delta\rho - \imath\gamma\omega_L \frac{\partial}{\partial\theta} - \frac{\omega_L^2}{4}\rho^2 + \frac{1}{\sqrt{(\rho + \rho_0)^2 + d^2}} + E\right]\Phi(\rho) = 0$$

$$\omega_L = \frac{\omega_c}{2}, \quad \gamma = \frac{m_h^* - m_e^*}{m_h^* + m_e^*}$$

$r_H = \sqrt{\frac{1}{\omega_L}}$

Numeric diagonalization of the Hamiltonian in different bases

Kaputkina, Lozovik. FTP 32(1998)1354

Indirect magnetoexciton with fixed layer widths in external magnetic field

$$\left[\Delta\rho + \frac{\partial^2}{\partial z^2} - \imath\gamma\omega_L\frac{\partial}{\partial\theta} - \frac{\omega_L^2}{4}\rho^2 + \frac{1}{\sqrt{(\rho+\rho_0)^2+d^2+z^2}} + E\right]\Phi(\rho) = 0$$

Eigensystem

$$\begin{aligned}
f_{nmk}(\rho,z) &= e^{\imath m\theta} L_n^{|m|}\left(\frac{\omega_L\rho^2}{2}\right) e^{-\frac{\omega_L\rho^2}{4}}\rho^{|m|}\left(\frac{n!}{\pi(n+|m|)!}\left(\frac{\omega_L}{2}\right)^{|m|+1}\right)^{\frac{1}{2}} \\
&\times \frac{\sin\frac{\pi k(z-d)}{2D}}{D^{1/2}} \\
E_{0nmk} &= 2\omega_L\left(n + \frac{|m|-\gamma m+1}{2}\right) + \frac{\pi^2k^2}{(2D)^2} \\
& n = 0,1,2,\dots;\ m = 0,\pm1,\pm2,\dots;\ k = 1,2,\dots
\end{aligned}$$

[KL05]

Effective mass of magnetoexciton

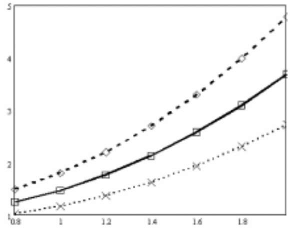
Ground state ($m = n = 0$)

$$M_{eff} = \frac{\exp\left(-\frac{\omega_c d^2}{2}\right)}{\int_{d^2}^{\infty} \exp\left(-\frac{\omega_c t}{2}\right) \frac{t^2 - 3\left(t + \frac{d^2}{4}\right)}{t^{5/2}} dt}$$

for $m = 0$ effective mass of magnetoexciton is positive;
for $m \neq 0$ effective mass of magnetoexciton can be negative -

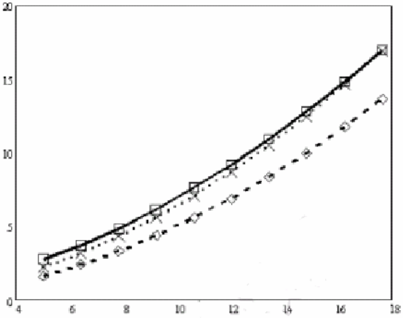
[KL05] Kaputkina, N.E. and Lozovik, Yu.E. *Physica E* **26**(2005)291

The dependencies of effective mass of magnetoexciton M_{eff}



vs. interlayer distance d, layer width $D = 1$ a.u. 1 – dotted line – $H = 2.5$ a.u., 2– solid line – $H = 3$ a.u., 3– dashed line –$H = 3.5$ a.u.

M_{ef}



vs. H. 1 – dashed line – $d = 1.2$; $D = 0.6$ 2 – dotted line – $D = d = 0.9$, 3– solid line – $d = 0.55$; $D = 1.25$

Indirect exciton in CQDs in magnetic field

$$\hat{H}_m = \frac{e}{2c}\left(\frac{A_e^2}{m_e^*} - \frac{2\imath\hbar\nabla_e A_e}{m_e^*} + \frac{A_h^2}{m_h^*} + \frac{2\imath\hbar\nabla_h A_h}{m_h^*}\right),$$

$$[\mu_h(\alpha_e - 1) - \mu_e(\alpha_h - 1)] \to 0, \alpha_1 \to \alpha_1' = \alpha_1 + \frac{\omega_c^2}{16},$$

Magnetic field

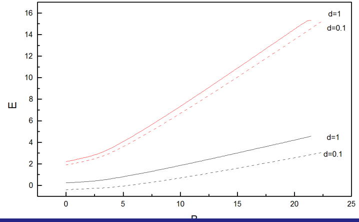
$$B \to \infty : E_r \to 2\sqrt{\alpha_2'}(2n + |m| + 1) - \gamma\omega_c m$$

$$d \to \infty : E_r \to 2\sqrt{\alpha_2'}(2n + |m| + 1) - \gamma\omega_c m - \frac{1}{d} + \frac{1}{4\sqrt{\alpha_2'}d^3}$$

$$d \to 0 : V_{nn'}^m \to -\left(\frac{n!n'!}{(n+|m|)!(n'+|m|)!}\alpha_2'\right)^{\frac{1}{2}} \times$$

$$\times \sum_{i=0}^{n}\sum_{j=0}^{n'}(-1)^{i+j}\binom{n+|m|}{n-i}\binom{n'+|m|}{n'-j}\frac{\Gamma\left(i+j+|m|+\frac{1}{2}\right)}{i!j!}$$

Indirect exciton in CQDs in magnetic field



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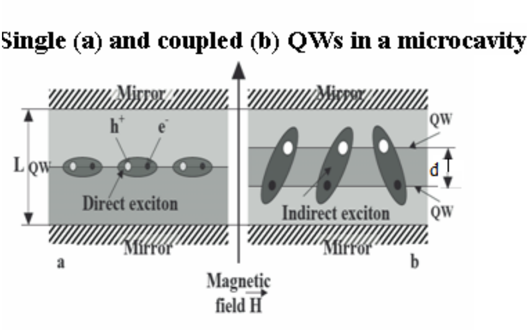
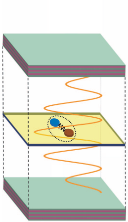
Polariton scheme

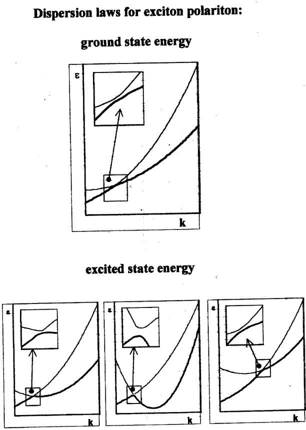




Polariton dispersion law

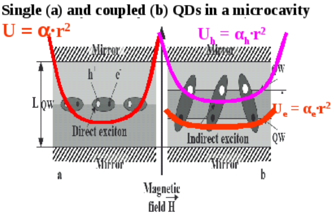
For excited levels with quantum number m not equal to zero (including the 1st excited level) effective mass of indirect magnetoexciton is negative for small magnetic momentum region. Yu.E.Lozovik , N.E.Kaputkina, phys.stat.sol.(b) 207, 147 (1998); Yu.E.Lozovik, A.M.Ruvinsky, Phys.Lett.A 227, 271 (1997); JETP 85, 983(1997); N. E. Kaputkina and Yu. E. Lozovik, Physica E 26, 291 (2005)

Dispersion laws for excited exciton polariton are nonmonotonous for certain range of parameters of coupled QWs, microcavity and magnetic field. Magnetic field controls the appearance and depth of the 2nd minimum and the relative depths of both minima.

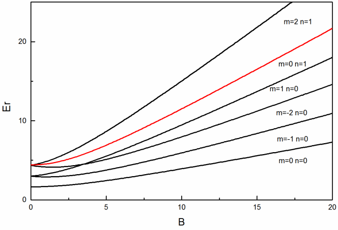


Effect of magnetic field on exciton polariton systems

Single and coupled QD in microcavity



Dependence of low-lying exciton relative motion energy levels on magnetic field



Magnetic field enhance the effective steepness of confining potential. This results in rearrangement of exciton and exciton polariton spectra. Kaputkina, Lozovik, Physica B. 403(2007)1537

At low temperatures spontaneous coherence and Kosterlitz-Thouless transition to superfluid state of exciton polaritons in the system of coupled quantum wells in microcavity or Bose-Einstein condensation of exciton polaritons in the system of coupled quantum dots in optical microcavity can take place. Magnetic field influence on the critical temperature can be non-monotonous for some region of control parameters for the competition of two mechanisms Kaputkina, Lozovik Phys.Stat.Sol.(c) 6(2009)20:

1. growth of the effective mass of magnetoexciton which leads to decreasing of critical temperature;
2. growth of effective parameter of confinement which increases the critical temperature and favors condensate formation.

Critical values T_c

$$T_c = \sqrt{\frac{3\alpha_{eff} Ns}{2M_{eff}}} \frac{h}{k_B \pi^2},$$

k_B is the Boltzman constant, s is spin degeneracy ($s = 2$ for bright excitons in GaAs QWs).

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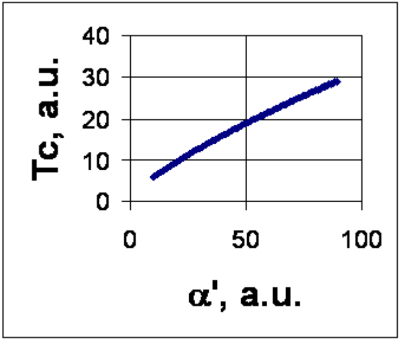
Influence of external magnetic field and confinement on Bose-condensation, and Kosterlitz-Thouless transition

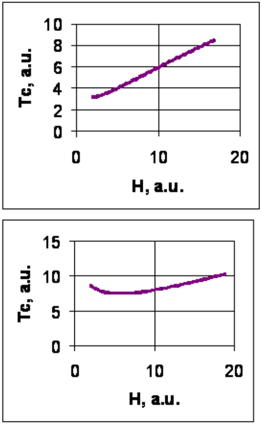
Growth of confining potential steepness favors condensate formation and increases the critical temperature



The dependence of critical temperature T_c on parameter of confinement steepness (α') for fixed magnetic field $H = 10$ a.u.

Influence of external magnetic field and confinement on Bose-condensation, and Kosterlitz-Thouless transition

Magnetic field influence on the critical temperature can be non-monotonous for some region of control parameters



$\alpha' = 10$

$\alpha' = 100$

The dependence of critical temperature T_c vs. magnetic field H, for fixed (α') Kaputkina, Lozovik Phys.Stat.Sol.(c) 6(2009)20

The cause of the non-monotonous effect of magnetic field is the competition of two mechanisms:

1. growth of the effective mass of magnetoexciton which leads to decreasing of critical temperature;
2. growth of effective parameter of confinement which leads to increasing of critical temperature;

Effects of external parameters on coupled QD in microcavity

- Magnetic field changes effective mass of magnetoexciton.
- Magnetic field increases effective steepness of confining potential in quantum dots also. This leads to the transformation of exciton energy spectrum in magnetic field.
- At low temperatures spontaneous coherence and Berezinskii-Kosterlitz-Thouless transition to superfluid state of exciton polaritons in the system of coupled quantum wells embedded in microcavity or Bose-Einstein condensation of exciton polaritons in the system of coupled quantum dots embedded in optical microcavity can take place.
- Growth of confining potential steepness favors condensate formation and increases the critical temperature.
- Magnetic field influence on the critical temperature can be non-monotonous for some region of control parameters.

Quantum dots for nanoelectronics

Using quantum dots for nanoelectronics

- Qubits
- Gates
- Quantum memory (with hierarchic access)
- Quantum neural nets

benefits from quantum parallelism

From Classical to Quantum Computation

- Classical information $\Rightarrow$ sequence of bits – classical systems with two allowed states (0,1)
- Miniaturization $\Rightarrow$ quantum effects, Heisenberg uncertainty principle $\Delta x \Delta p \sim \hbar$.
  Localization of electron within $\Delta x \sim 10^1$nm demands momentum transfer of $\frac{\hbar}{2\Delta x}$, e.g. velocity $v \sim 10^5 \mathrm{m/s}$ and MeV energy.
- Interaction of electron with environment at $T \sim 300$K implies the energy transfer of $k_B T \sim 25$ meV
- Single electron access $\Rightarrow$ the probability of a definite state of logical bit $P < 1$, constrained by quantum effects.

Bits

Classical bits

$\Longrightarrow$

Qubits

Quantum bits

Qubits on quantum dots

Quantum dots – artificial atoms

Typical size $\sim 10^1$nm
Qubits can be implemented on charge, electron spin states or excitonic states in the pairs of coupled QD

Advantages of spin degrees of freedom

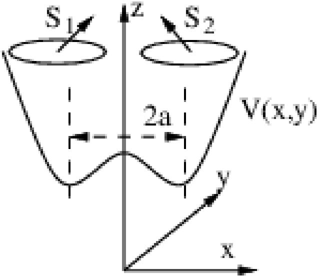
Spin relaxation time of the excess electron $\sim 10^{-3}$s
Spin decoherence time $\sim 10^{-6}$ s

Quantum dots are prospective as qubits

New possibilities

– access to collective states of quantum dot groups, instead of separate access to each spin
– hierarchic coding in quantum memory

Qubit pair on QD



Two coupled one-electron quantum dots separated by the distance $2a$ form a quantum gate. Magnetic field B is applied along the z direction. Electric field in x direction

Hamiltonian of QD pair

$$H = H_{kinetic} + H_{potential} + H_{Zeeman} + H_S,$$

$$H_{potential} = V(x,y) + \frac{e^2}{\varepsilon|r_1 - r_2|} + e\sum_{i=1}^{2} x_i E$$

$$H_{Zeeman} = g\mu_B \sum_i \boldsymbol{B}\cdot\mathbb{S}_i,$$

H_S – Heisenberg Hamiltonian

$$V(x,y) = \frac{m\omega_0^2}{2}\left[\frac{\left(x^2 - a^2\right)^2}{4a^2} + y^2\right].$$

X.Li et al. *Science* 301(2003)809

Decoherence

Long wave fluctuations of magnetic field does not affect the spin coherence if their length is greater than QD magnetic length ($\sim 10^1$nm) . Spin-orbital coupling may be neglected

$$H_{SO} = \frac{\omega_0^2}{2mc^2} \mathbb{L} \cdot \mathbb{S},$$

since $H_{SO}/\hbar\omega_0 \sim 10^{-7}$. DiVincenzo & Loss. *Superlatt. and Microstruct.* 23(1998)419 spin-spin interaction is proportional to $1/d^3$.

The interaction force may be tuned by making the sequence aperiodic.

Kaputkina, Lozovik, Muntyanu, Vekilov *J. Phys. Conf. Ser.* 226(2010)012028; Korotaev, Vekilov, Kaputkina JETP 113(2011)692

Dipole interaction between qubit spins and environment can be estimated as: $(g\mu_B)^2/a_B^3 \approx 10^{-9}$ meV for GaAs.

The hyperfine interaction with nuclear spins can be strongly suppressed either by dynamically polarizing the nuclear spins, or by applying magnetic field

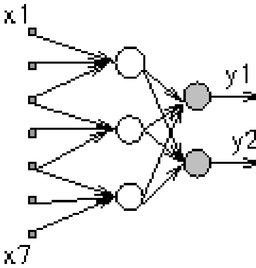
Addressing different spin states of the whole system, rather than different quantum bits, can allow for a "flexible" memory elements (say, on aperiodic sequences of quantum dots). The price paid for such flexibility is the spectroscopic problem to distinguish reliably the spin states of quantum system containing 2, 4 and more 2^M spins.

Altaisky, Kaputkina IJQI 10(2012)1250026

Since the size of the physical support of the group of spins, manipulated spectroscopically, say a group of excess electrons in quantum dots, is comparable to the size of single qubit of the same nature, our method possibly provides a new way of miniaturization of memory elements on nanoscale heterostructures.

Quantum neural networks

- Quantum neural networks are the nonunitary alternative to unitary quantum computing which benefits rather than suffers from interaction with environment, exactly as human brain does.
- Quantum neural networks are presently built on SQUIDs Johnson et al. Nature 473(2011)194 , but hypothetically can be constructed on quantum dots Behrman et al. Inf. Sci. 128(2000) 257.
- Alternative to SQUID based quantum Hopfield network might be one made of quantum dots M.V.Altaisky, N.E.Kaputkina, V.A. Krylov *PEPAN* 45(2014) 1013-1032



Quantum dot controlled by magnetic field can be tried as artificial neuron. Such neurons can be arranged as an array on e.g. GaAs substrate

N.E. Kaputkina and Yu.E. Lozovik, *Dimensional effects and magnetic field influence on excitons in coupled quantum dots and coupled quantum wells*, Physica E **26** (2005), no. 1, 291–296.

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M.V.Altaisky, N.E.Kaputkina, V.A. Krylov *PEPAN* 45(2014) 1013-1032

- Нейронные сети

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- Квантовые нейронные сети на основе квантовых точек
- Моделирование взаимодействующих КТ, связанных с общим термостатом

1943 McCulloch and Pitts proposed the mathematical model of neuron [MP43]

1967 Riccardi and Umezawa proposed the quantum model of human memory

1970 V.Chavchanidze proposed a model for quantum phase calculations in human brain [Чав70]

1982 R.Feynman proposed the idea of quantum simulator [Fey82]

1992 F.Beck and J.Eccles emphasized the role of quantum tunneling in activation of neuron [BE92]; Deutsch and Josza presented the concept of quantum computational network [DJ92]

1995 S.Kak proposed the concept of quantum neural network [Kak95]

1997 T.Nitta proposed a complex-valued neural net [Nit97]; A.Vlasov proposed quantum associative memory [Vla97]

Historical notes

2000 Kouda, Matsui, Nishimura proposed qubit network [KMN00]; E.Behrman et al. proposed QNN on quantum dots [BNS$^+$00]

2001 M.Altaisky proposed quantum perceptron [Alt01]

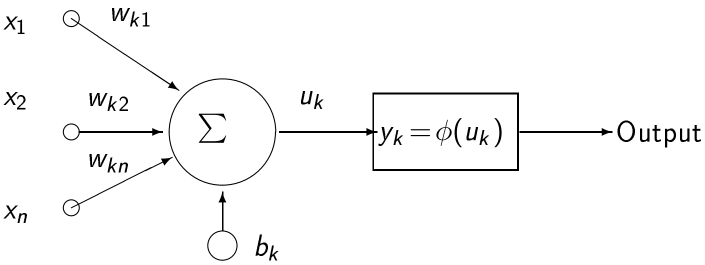
2006 China launched the program for QNN for space technology [ZJD06, ZD07]

2010 D-wave systems Inc. put the first quantum computer 'Rainer' (D-wave One, 128 qubit) on the market [JAG$^+$11] (\$ 1.0×10^{10})

2012 NASA launched the program for quantum network artificial intellegence [Sa12]

2013 The 512 qubit Quantum network computer 'Vesuvius' (D-wave Two) was tested (\$ 1.6×10^{10})

Модель нейрона



$$u_k = \sum_{j=1}^{N} w_{kj} x_j + b_k, \qquad y_k = \phi(u_k)$$

Choice of sigmoid function $\phi(u) =$

$$\theta(u), \qquad tanh(u), \qquad \frac{1}{1+e^{-u}}$$

Application of neural networks

- Image recognition – technical vision
- Associative memory
- Artificial intellegence for autonomous missions
- Technical safety systems
- Classification problems
- Complex optimization problems
- Medical diagnostics
- Time series forecast

- Learning with teacher

$$\Delta w_{ij} = \eta y_j d_i.$$

  d_i is desired output of i-th neuron
- Self-organized learning
  Hebbian rule for biological neuron [Heb49]:

$$\Delta w_{ij} = \eta y_j y_i$$

- Hopfield network:

$$H = \sum J_{ik} s_i s_k - h_k s_k$$

The goal of the ANN learning with teacher is to minimize the 'error energy'

$$E = \frac{1}{2} \langle \sum_{i=1}^{M} (y_i^k - d_i)^2 \rangle$$

This can be done by gradient descend or by other methods

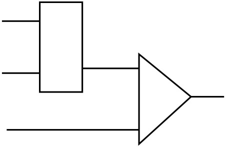
Classical information

Bit

Physical system that may be in either of distinct physical states "0" or "1","off" or "on",$\downarrow$ or $\uparrow$

$$0+0=0, 0+1=1, \quad 1+1=10$$
$$0*0=0, 0*1=0, \quad 1*1=1$$
$$\neg 0=1 \qquad \neg 1=0$$

Logical circuits



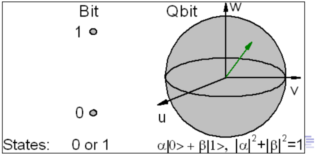
Quantum information

Qubit =quantum bit

$$|\psi\rangle = c_0|0\rangle + c_1|1\rangle,$$
$$c_0, c_1 \in \mathbb{C}, \quad |c_0|^2+|c_1|^2=1$$

Bloch sphere

$$|\theta,\phi\rangle = \cos\frac{\theta}{2}|\uparrow\rangle + e^{\imath\phi}\sin\frac{\theta}{2}|\downarrow\rangle,$$
$$0\le\theta\le\pi; 0\le\phi\le 2\pi.$$



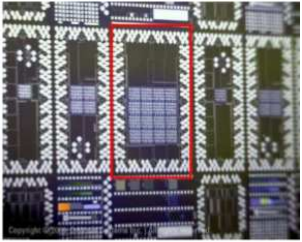
Quantum annealing machines D-wave Systems Inc.

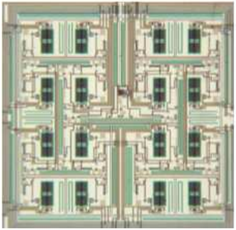
M.Johnson et al. *Nature* **473**(2011)194

The first scalable quantum computer was constructed by D-Wave Systems Inc. It is capable of solving exponentially difficult minimization problems

$$H_P = -\sum_{i=1}^{N} h_i \sigma_i^z + \sum_{i,j=1}^{n} J_{ij} \sigma_i^z \sigma_j^z$$

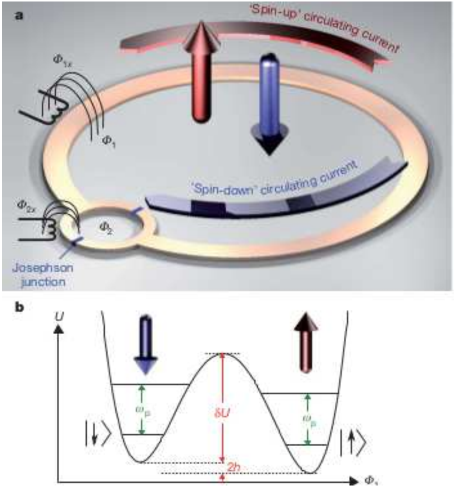
in polynomial time. The 'spins' σ_i^z are implemented in Superconducting Quantum Interference Devices. The connection matrix J_{ij} is implemented by inductive couplings. The superconducting circuit technology is used.





128 qubit SQUID processor.
From arxiv.org:1204.2821

SQUID flux qubit



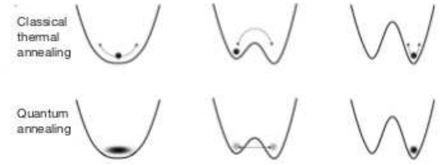
Redrawn from M.Johnson et al.
Nature **473**(2011)194

Realization of quantum annealing

- All 'spins' are initialized in X direction at $t = 0$

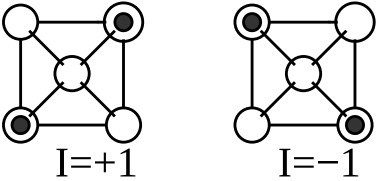
$$H(t) = -\Gamma(t)\sum_{i=1}^{n}\Delta_i(t)\sigma_i^x + \Lambda(t)\left[-\sum_{i=1}^{n}h_i\sigma_i^z + \sum_{i,j=1}^{n}J_{ij}\sigma_i^z\sigma_j^z\right]$$

- The transverse magnetic field is adiabatically turned off $\Gamma(t) \to 0$ with simultaneous increase of $\Lambda(t) \to 1$



Redrawn from M.Johnson et al. *Nature* **473**(2011)194

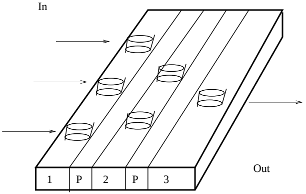
In 2000 E.C.Behrman et al. proposed a model of quantum neural network where the nonlinearity is attained by means of interaction of quantum dot array with the substrate phonons. Each neuron was designed as a "molecule" of 5 quantum dots



Недостатки: управление такой сетью гипотетически осуществляется за счет измения спектра фононов, что практически нереализуемо.

Quantum dot network with dipole-dipole interaction

The Behrman network control is completely nonlocal and unstable to thermal fluctuations. For this reason we propose a network made of QD array with dipole-dipole interaction between neighboring dots on the GaAs substrate controlled by plasmons.



$$\hat{H}(t) = \sum_i K\sigma_i^x + \sum_i \Delta_i(t)\sigma_i^z + \sum_{i,\alpha} \lambda_i^\alpha x_\alpha(t)\sigma_i^z +$$
$$+ \sum_\alpha \frac{m\dot{x}_\alpha^2(t)}{2} + \sum_\alpha \frac{m\omega_\alpha^2 x_\alpha^2(t)}{2} + \sum_{m\neq j} \frac{\vec{d}_m \cdot \vec{\sigma}_m^\dagger \otimes \vec{d}_j \cdot \vec{\sigma}_j}{\varepsilon_{mj}(t) R_{mj}^3}$$

Model Hamiltonian for two QDs with d-d interaction

$$H = \sum_{i=1}^{2} \frac{\delta_i}{2}(\sigma_z^{(i)} + 1) + \sum_{i=1}^{2} \frac{K_i}{2}\sigma_x^{(i)} + \sum_{i\neq j} J_{ij}\sigma_+^{(i)}\sigma_-^{(j)}$$
$$+ \sum_{a,i} g_a x_a |X_i\rangle\langle X_i| + H_{phonon},$$

$\delta_i = \Delta_i - \omega$ is detuning, Δ_i is the energy gap between the ground and the first excited state of the i-th QD; K_i is a coupling to an external driving field, J_{ij} is the dipole-dipole coupling, constructed in analogy to the dipole-dipole interaction of atoms [CF78, JQ95].

$$\sigma_z^{(i)} = |\mathrm{X}_i\rangle\langle\mathrm{X}_i| - |0_i\rangle\langle 0_i|, \sigma_x^{(i)} = |0_i\rangle\langle\mathrm{X}_i| + |\mathrm{X}_i\rangle\langle 0_i|,$$
$$\sigma_+^{(i)} = |\mathrm{X}_i\rangle\langle 0_i|, \sigma_-^{(i)} = |0_i\rangle\langle\mathrm{X}_i|.$$

The phonon modes x_a are assumed to interact only to the excited states $|X_i\rangle$

Phonon bath parametrization

The free phonon Hamiltonian is

$$H_{Ph} = \sum_a \frac{p_a^2}{2m_a} + \frac{m_a \omega_a^2 x_a^2}{2},$$

The phonons in GaAs substrate are assumed to have the spectral density

$$J(\omega) = \frac{\pi}{2} \sum_a \frac{g_a}{m_a \omega_a} \delta(\omega - \omega_a) \approx \alpha \omega^3 \exp(-(\omega/\omega_c)^2),$$

This form for $J(\omega)$ is the excellent agreement between experiment and theory of the interaction of single quantum dot with phonon bath [RFS$^+$10, RGB$^+$10, MDG$^+$11, Dat12, DPW12, Dat13].

$$\alpha = \frac{(D_e - D_h)^2}{4\pi^2 \rho \hbar v_s^5} = 0.032\mathrm{ps}^2 \quad \text{for GaAs}, \quad \omega_c = \frac{\sqrt{2} v_s}{d}$$

In our studies we used the quasi-adiabatic propagator path integral (QUAPI) technique [MM95a, MM95b] for the solution of the von Neumann equation for the density matrix $\rho(t)$, which describes the evolution of the above described pair of interacting quantum dots:

$$\dot{\rho} = \mathrm{tr}_{\mathrm{Ph}} \left(-\frac{\imath}{\hbar}[H, \rho_{\mathrm{tot}}] \right),$$

with the initial condition:

$$\rho_{\mathrm{tot}}(0) = \rho(0) \otimes \frac{e^{-\beta H_{\mathrm{Ph}}}}{\mathrm{tr}\left(e^{-\beta H_{\mathrm{Ph}}}\right)}.$$

where for the particular case of two interacting QDs

$$\rho(0) = |\psi(0)\rangle\langle\psi(0)| \; , \; |\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|0X\rangle + |X0\rangle\right)$$

Quasiadiabatic path integral

The time dependence of the reduced density matrix of the QD system is given by the Feynman integral

$$\langle s_N^+|\rho(t)|s_N^-\rangle = \int \left(\prod_{m=0}^{N-1} \langle s_{m+1}^+|e^{-\frac{\imath\Delta t}{\hbar}H_{OQS}}|s_m^+\rangle\langle s_m^-|e^{\frac{\imath\Delta t}{\hbar}H_{OQS}}|s_{m+1}^-\rangle\right)\times$$

$$\times \qquad \langle s_0^+|\rho(0)|s_0^-\rangle I\left(\{s_m^\pm\}^N;\Delta t\right)\prod_{m=0}^{N-1} ds_m^+ ds_m^-,$$

where s_m^+ (s_m^-) denotes the state of the OQS at $t_m = m\Delta t$ on the time-forward (time-backward) propagation. The discretized bath influence functional is equal to

$$I\left(\{s_m^\pm\}^N;\Delta t\right) = e^{-\sum_{mm'}(s_m^+ - s_m^-)(\eta_{mm'}s_{m'}^+ - \eta_{mm'}^* s_{m'}^-)},$$

with $\eta_{mm'}$ being the discretized version of $\alpha(t)$ given in [Dat13] according to quasi-adiabatic propagator path integral method [MM94].

In our simulations we used the representation of QUAPI method given in Vagov,Croitoru,...,Kuhn. *PRB* **83**(2011) 094303][VCG⁺11] The total evolution time t is divided into N time slices $t_n = \epsilon n, \epsilon = t/N$. The final density matrix at time $t_N = t$ is given by

$$\rho_{\alpha_N,\beta_N} = e^{\imath t(\hat{\Omega}_{\beta_N\beta_N} - \hat{\Omega}_{\alpha_N\alpha_N})} \sum_{\{\alpha_n,\beta_n\}} \prod_{n=1}^{N} M_{\alpha_n}^{\alpha_{n-1}} M_{\beta_{n-1}}^{\beta_n *} \prod_{n'=1}^{n} e^{S_{nn'}} \rho_{\alpha_0\beta_0},$$

where $\hat{\Omega} = \mathrm{diag}(0, \Delta, \Delta, 2\Delta)$ is the diagonal part of the system Hamiltonian without bath. The evaluation is performed using the method of augmented density matrix evaluation [MM95a, MM95b]: $\overline{R}_n = T_n \overline{R}_{n-1}, R_0 = \rho(0)$ where the augmented density matrix $\overline{R}_n$ coincides with the density matrix of the system R_n for all discrete time instants less or equal to the memory length n_c, or is truncated by the last time instant $n - n_c - 1$.

Augmented density matrix evolution

$$R_n = T_n R_{n-1}, n \leq n_c$$

Starting from $n = n_c + 1$ all paths with coinciding states except for the earliest time state at $n - n_c - 1$ are summed up over that state to keep the non-increasing record length of n_c time instants. The iteration operator has the form

$$T_n = M_{\alpha_n}^{\alpha_{n-1}} M_{\beta_{n-1}}^{\beta_n *} \exp\left\{ \sum_{n'=\max(1,n-n_c)}^{n} S_{nn'} \right\}$$

The system rotation operator has the form

$$M_{\alpha_n}^{\alpha_{n-1}} \equiv \langle \alpha_n | e^{-\imath\epsilon \hat{M}(t_n)} | \alpha_{n-1} \rangle \equiv M(\alpha_n, \alpha_{n-1}) \qquad (1)$$

$\alpha, \beta = \overline{0,3}$ are the indexes of the quantum state of the system. For the two qubits these states are $00, 0X, X0, XX$.

$$M_{\beta_{n-1}}^{\beta_n *} \equiv M^\dagger(\beta_n, \beta_{n-1}),$$

where $\hat{M}$ is the non-diagonal part of the system Hamiltonian

'Action' functional

α_n, β_n are the time forward (backward) state variables.
$\gamma = (0,1,1,2)$ is the coupling of the states (00,X0,0X,XX) to phonons. The 'action' $S_{nn'}$ is given by the equation (A34) of [VCG$^+$11]:

$$S_{nn'} = -K_{\alpha'_n \alpha_n} - K^\dagger_{\beta'_n \beta_n} + K^\dagger_{\beta'_n \alpha_n} + K_{\alpha'_n \beta_n} \tag{2}$$

For real-valued phonon coupling this gives

$$\begin{aligned} S_{n\neq n'} &= (\gamma(\alpha'_n) + \gamma(\beta'_n))(\gamma(\beta_n) - \gamma(\alpha_n))K(n'-n), \\ S_{nn} &= (\gamma(\beta_n) - \gamma(\alpha_n))(\gamma(\alpha_n)K(0) - \gamma(\beta_n)K^*(0)) \end{aligned}$$

where $K^*(-m) = K(m), \quad m \neq 0$ was used.

Bath correlation function

$$K_{m\neq 0} = \int_{m\epsilon}^{(m+1)\epsilon} d\tau \int_0^{\epsilon} d\tau' \Gamma(\tau-\tau') =$$
$$= 2\int_0^{\infty} d\omega \frac{J(\omega)}{\omega^2}[1-\cos(\omega\epsilon)]\{\cos(\omega\epsilon m)\coth\frac{\omega\beta}{2} - \imath\sin(\omega\epsilon m)\}$$
$$K_{m=0} = \int_0^{\epsilon} d\tau \int_0^{\tau} d\tau' \Gamma(\tau-\tau') =$$
$$= \int_0^{\infty} d\omega \frac{J(\omega)}{\omega^2}\{[1-\cos(\omega\epsilon)]\coth\left(\frac{\omega\beta}{2}\right) - \imath\epsilon\omega + \imath\sin(\epsilon\omega)\}$$

We introduce dimensionless variables:
$y = \frac{\omega}{\omega_c}, \quad \epsilon' = \epsilon\omega_c, \quad \beta' = \frac{\hbar\omega_c}{k_B T}$
The integrals were evaluated analytically using the approximation [JCS02, RE14]: $\coth(y) \approx 1 + 2e^{-2y} + 2e^{-4y} + \frac{e^{-5y}}{y}$

Quantum dot parameters

Ramsay et al. *Phys. Rev. Lett.* **105**(2010)177402;
McCutcheon et al. *Phys. Rev. B* **84**(2011)081305;
Dattani N.S. *Comp. Phys. Comm.* **184**(2013)2828

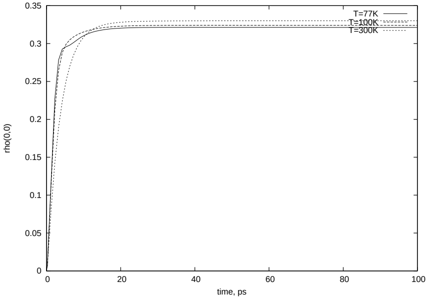
$$m_* = 0.067 m_e, \rho_{GaAs} = 5.37 g/cm^3, v_s = 5.11 \cdot 10^5 cm/s,$$
$$\epsilon = 10.0, a_0 = 3.94 nm, E_0 = 36.5 meV$$

$d = 3.3$nm, $L = 10$nm
Dipole coupling $J = 0.595 ps^{-1}$
Driving field $K = 0.476 ps^{-1}$
Energy gap
$\Delta = 158 ps^{-1} \approx 104 meV$
Cutoff frequency $\omega_c = 2.2 ps^{-1}$

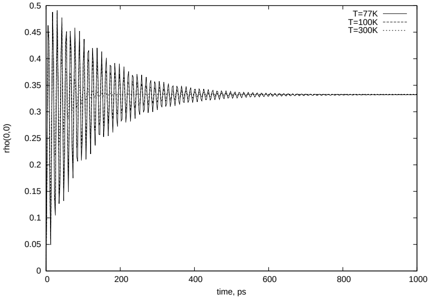
d=33nm, L = 50nm
Dipole coupling $J = 0.476 ps^{-1}$
Driving field $K = 4.76 ps^{-1}$
Energy gap
$\Delta = 1.58 ps^{-1} \approx 1.04 meV$
Cutoff frequency $\omega_c = 0.22 ps^{-1}$

Evolution of the ground state

Evolution of the density matrix element $\rho(0,0)$ with the initial condition $0X + X0$



for d = 3.3nm InGaAs/GaAs QDs with $L = 10$nm



for d = 33nm InGaAs/GaAs QDs with $L = 50$nm

Being written in the 'magic basis' of Bell states the asymptotics of evolution corresponds to the spread of the state e_3 into the equally weighted triplet (e_1, e_2, e_3). The asymptotic density matrix ρ written in magic basis is $\rho(+\infty) = \mathrm{diag}(1/3, 1/3, 1/3, 0)$.

The basic vectors represent the phase modified Bell basis

$$|e_1\rangle = \frac{1}{\sqrt{2}}(|XX\rangle + |00\rangle), \qquad |e_2\rangle = \frac{\imath}{\sqrt{2}}(|XX\rangle - |00\rangle)$$
$$|e_3\rangle = \frac{\imath}{\sqrt{2}}(|X0\rangle + |0X\rangle), \qquad |e_4\rangle = \frac{1}{\sqrt{2}}(|X0\rangle - |0X\rangle).$$

The Werner state

$$W_{5/8} = \frac{5}{8}|e_4\rangle\langle e_4| + \frac{1}{8}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| + |e_3\rangle\langle e_3|\right)$$

is a mixed state, which is $5/8$ singlet and $3/8$ triplet.
Such states can be produced by using the equal ammounts of singlets and uncorrelated spins [Wer89] Werner RF, **PRA** 40(1989)4277

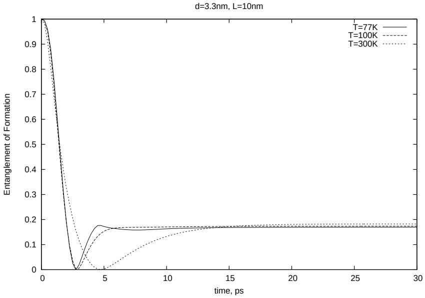
For a mixed state of a bipartite system the entanglement of formation is defined as a minimal possible entanglement over all quantum ensembles representing the mixed state [BDSW96]. The entanglement of formation of the calculated density matrix is evaluated in the Bell basis using the following procedure [BDSW96, HW97]. The four eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ of the auxiliary matrix

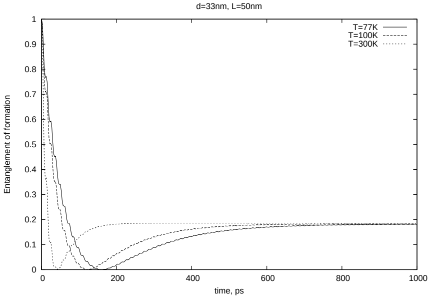
$$R(\rho) = \sqrt{\sqrt{\rho}\rho^*\sqrt{\rho}},$$

where ρ^* denotes the complex conjugation, are used to evaluate the *concurence* $C = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$. The entanglement of formation is then given by

$$E(\rho) = H\left(\frac{1}{2} + \frac{1}{2}\sqrt{1 - C^2}\right), \quad H(x) = -x\log_2 x - (1-x)\log_2(1-x).$$

Time evolution of entanglement





Conclusion

• QD systems can be controlled by external electric and magnetic field. This allows to tune electronic spectra and transport.

• In certain domain of control parameters the magnetic field effect on BEC of polaritons in trap occures to be nonmonotonous. Magnetic field allows for control polariton splitting, polariton resonance, polariton dispersion. It is prospective for optoelectronics.

• QDs are also promising for nanoelectronics, for qubits and for the quantum gates. The hierarchic access to memory registers can be built on spin degrees of freedom of QD arrays, proving an extra reliability of information storage by virtue of controlling spin of the whole array and its sub-blocks

• QD arrays controlled by magnetic field can be tried for quantum neural networks. We present the results of the simulation on a QNN based on QDs using the numerical path integral calculation. In a proposed implementation of QNN using an array of single-electron QDs with dipole-dipole interactions the coherences are shown to survive up to nanosecond time and up to the liquid nitrogen temperature of 77K and above.

THANK YOU FOR YOUR ATTENTION !!!

Perspectives

- Low energy consuming QNN
- Compatibility with optical devices

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