Туапсе-2015

Динамика коррелированных систем: возможности и проблемы численного описания

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Элементарный пример интегрирования по гауссовым переменным

Классический аналог коррелированной примеси в некоррелированном окружении:
нелинейный осциллятор, связанный с линейными.

$$\ddot{x} + W'(x) = y$$

$$\ddot{y} + \omega_0^2 y = x$$

$$y_\omega = \frac{x_\omega}{\omega_0^2 - \omega^2} \equiv \Delta_\omega x_\omega$$

или

$$y_t = \int \Delta_{t-t'} x_{t'} dt'$$

$$\ddot{x}_t + W'(x_t) = \int \Delta_{t-t'} x_{t'} dt'$$

+: удается исключить все некоррелированные переменные

-: задача оказывается негамильтоновой

Фейнмановская формулировка

Действие

$$S = \int \left(\frac{m\dot{x}(t)^2}{2} - U(x(t)) \right) + \int \Delta(t-t')x(t)x(t')dtdt'$$

Оператор эволюции

$$W = \int e^{-iS[x(t)]}dt$$

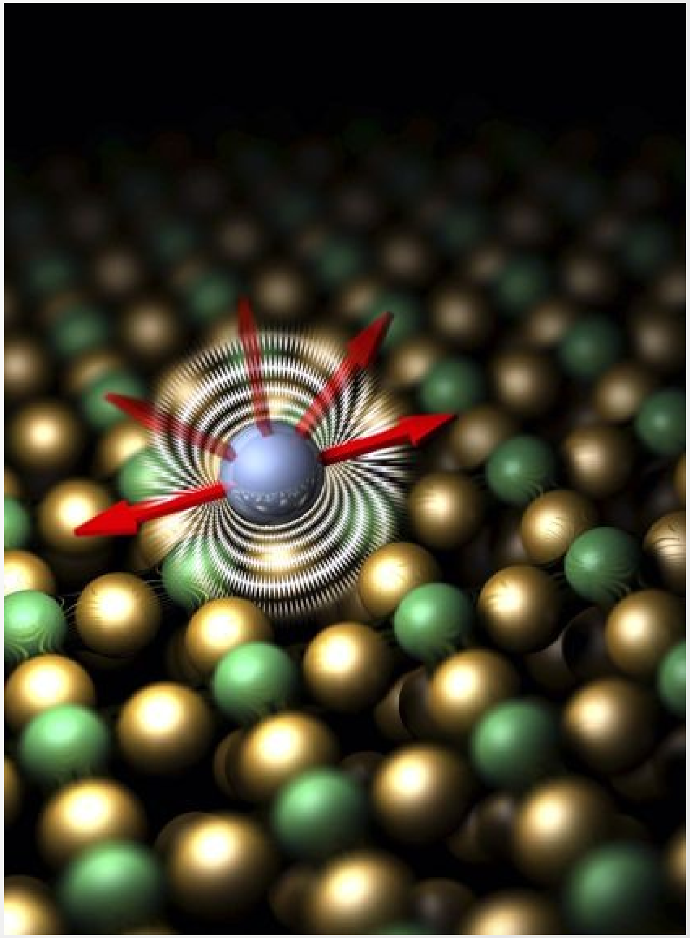
Стат. сумма

$$Z = \int e^{-S[x(\tau)]}d\tau$$

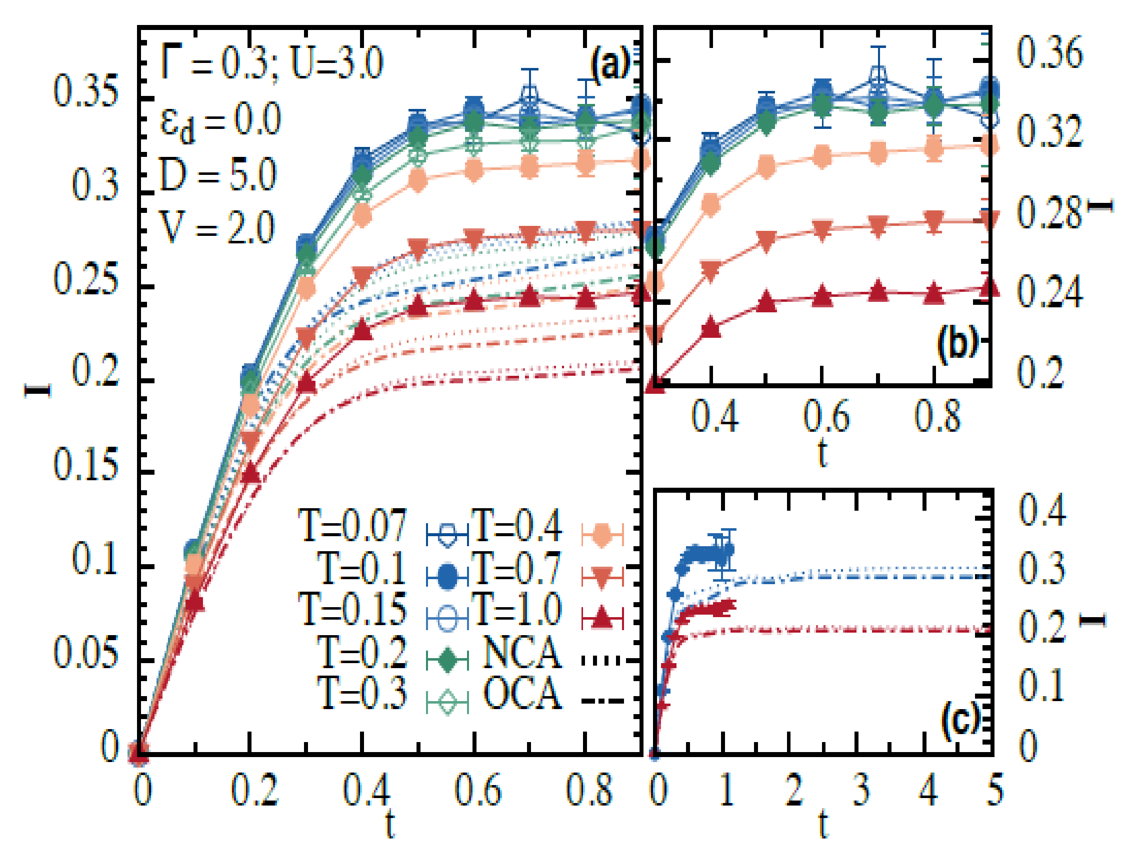
Действие примесной задачи

$$S = S_{at} + \int \Delta_{t-t'} c_t^{\dagger} c_{t'} dt dt'$$

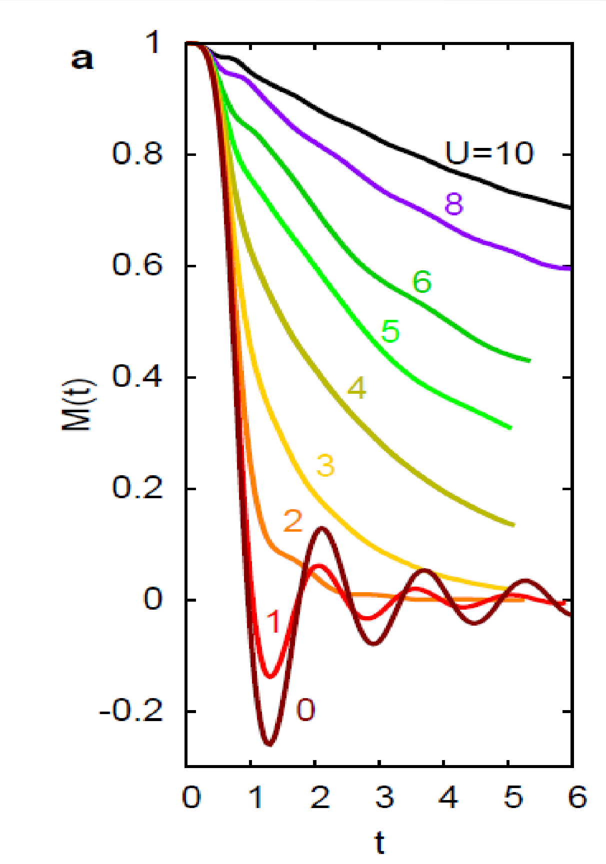
Решение задачи с запаздыванием
требует использования
численных методов



Recent resuls (other groups)

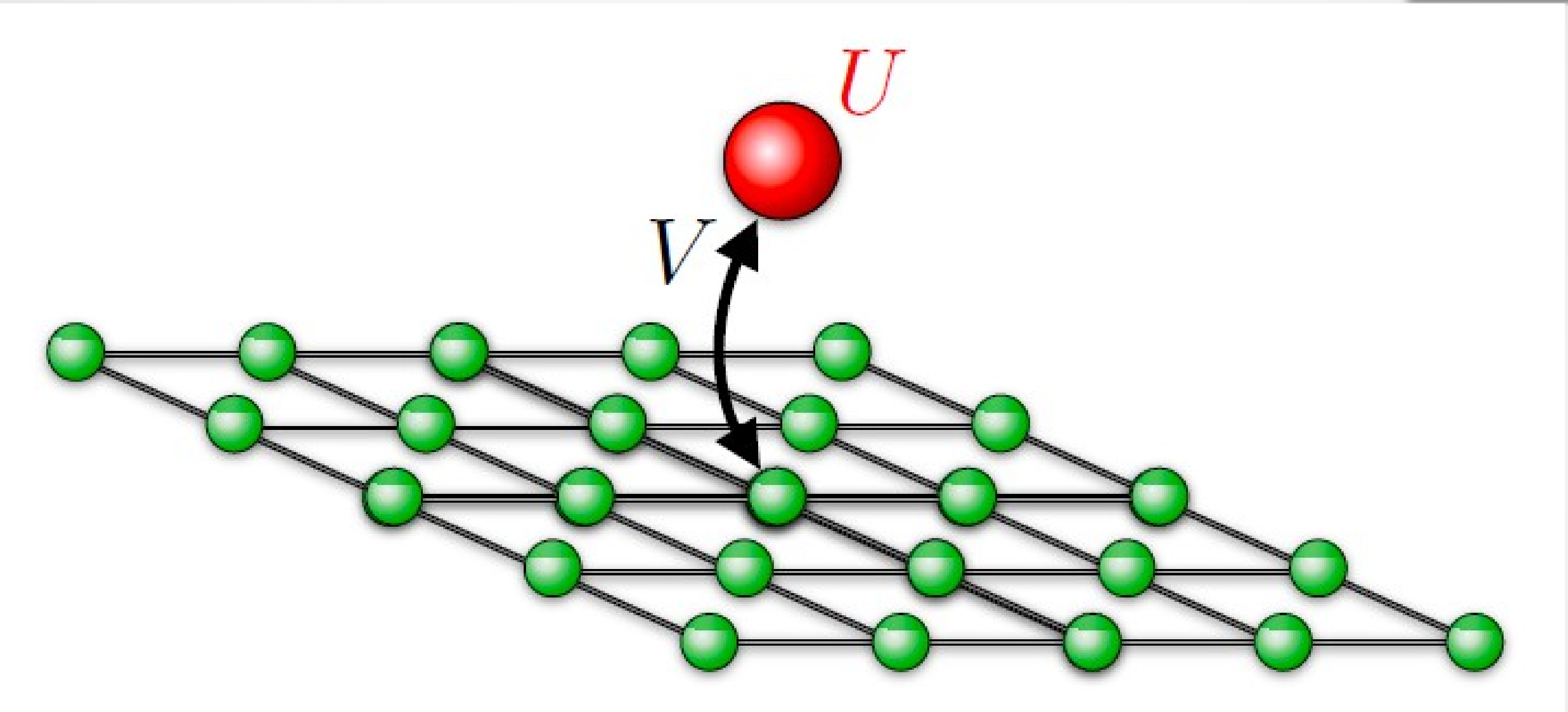


A.Antipov, Q. Dong, E.Gull
arXiv: 1508:06633



H. Strand, M. Eckstein, Ph. Werner
arXiv 1405.6941

B-SIAM: definitions

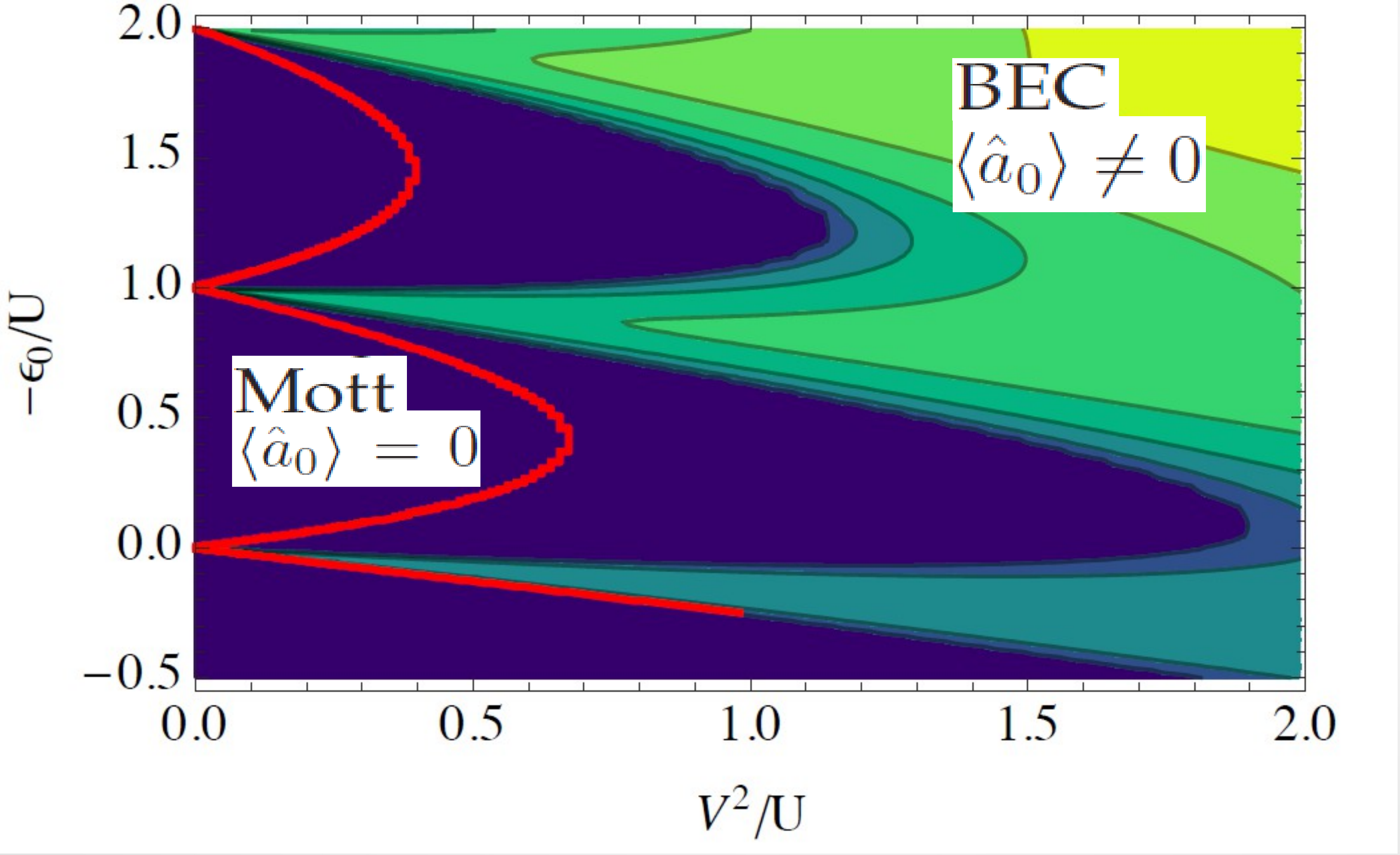


$$H = \varepsilon_0 a_0^\dagger a_0 + \frac{1}{2} U \hat{n}_0(\hat{n}_0 - 1) + \sum_k \varepsilon_k b_k^\dagger b_k - \sum_k V_k (b_k^\dagger a_0 + a_0^\dagger b_k)$$

$$\varepsilon_k = 2 - \cos k_x - \cos k_y$$

B-SIAM: quantum phase transitions in a 0d system

NRG calculations: R. Bulla et al, 2010



b-SIAM: mean-field equations

$$H_{imp} = \varepsilon_0 n_0 + \frac{1}{2} U n_0 (n_0 - 1) - (\lambda a_0^\dagger + \lambda^\dagger a_0)$$

$$\lambda = \Delta < a_0 >$$

Statics:

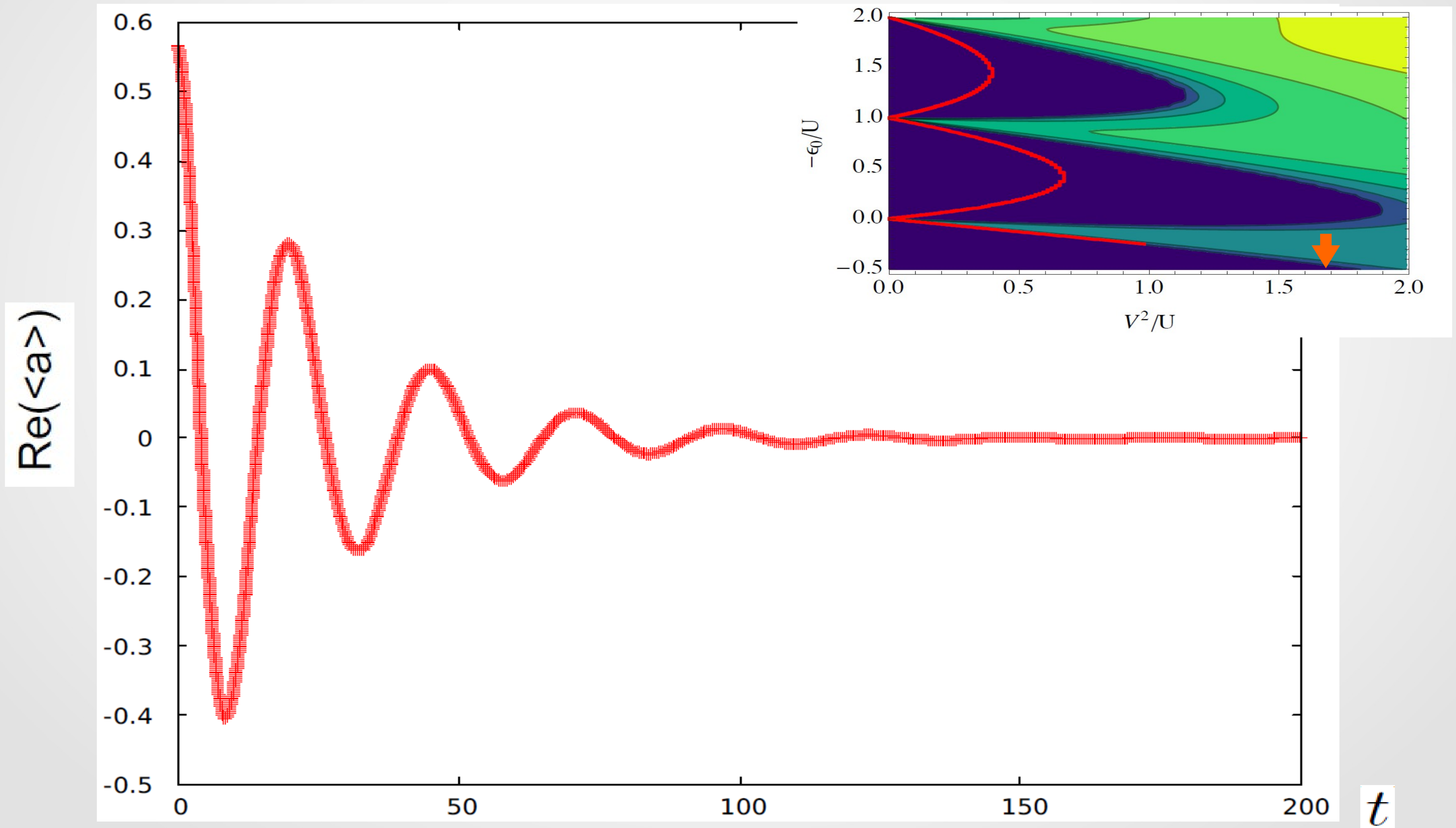
$$\Delta = \sum_k \frac{V^2}{\varepsilon_k}$$

Dynamics:

$$\hat{\Delta} = \sum_k \frac{V^2}{-i\partial_t + \varepsilon_k}$$

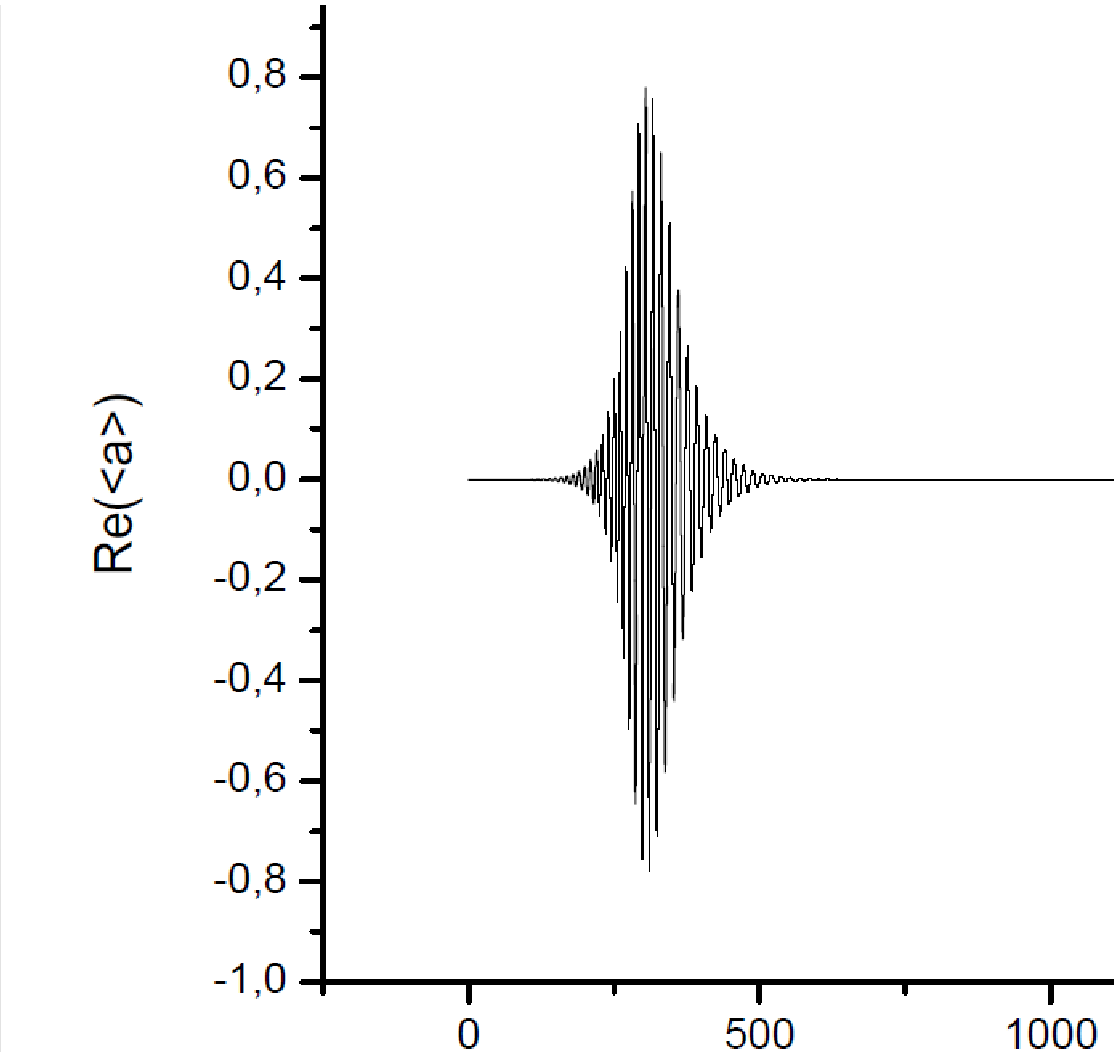
$$\hat{\Delta} < a > (t) = V^2 \left(\sum_k \frac{e^{-i\varepsilon_k t}}{\varepsilon_k} \right) < a_{t=0} > -iV^2 \int_0^t \left(\sum_k e^{-i\varepsilon_k (t-t')} \right) < a_{t'} > dt'$$

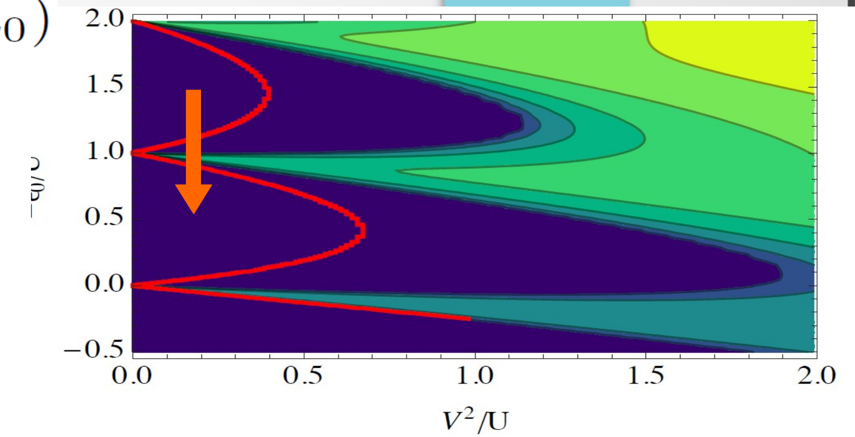
b-SIAM: quench from BEC to Mott phase



b-SIAM: quench from Mott to Mott phase

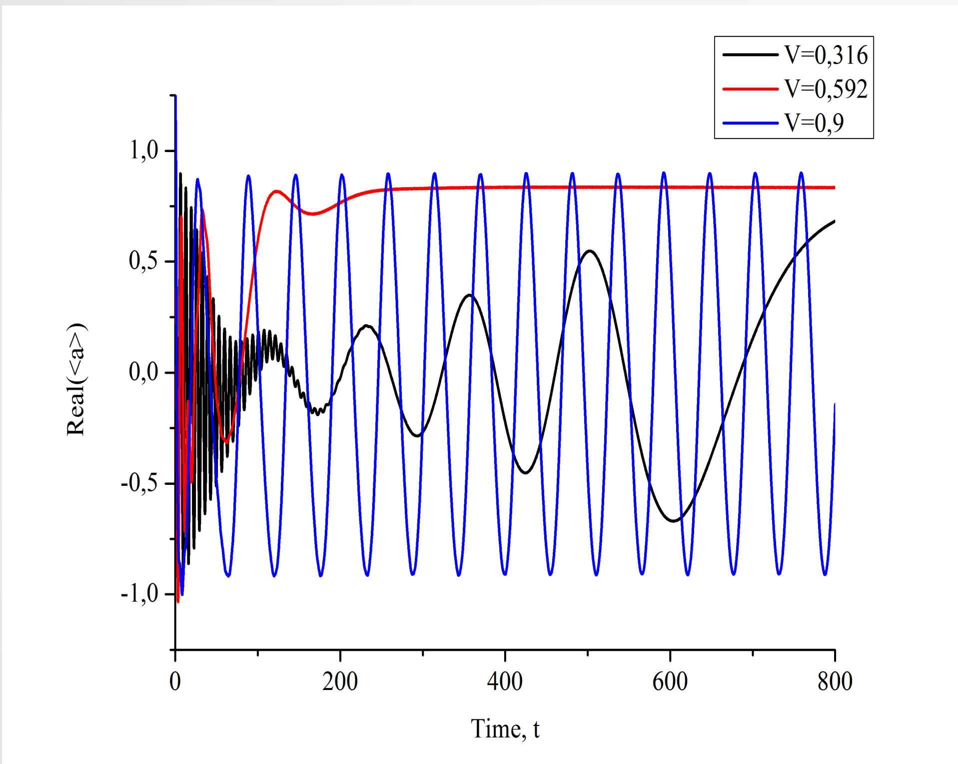
$$H_{imp} = \varepsilon_0 n_0 + \frac{1}{2} U n_0 (n_0 - 1) - (\lambda a_0^\dagger + \lambda^\dagger a_0)$$

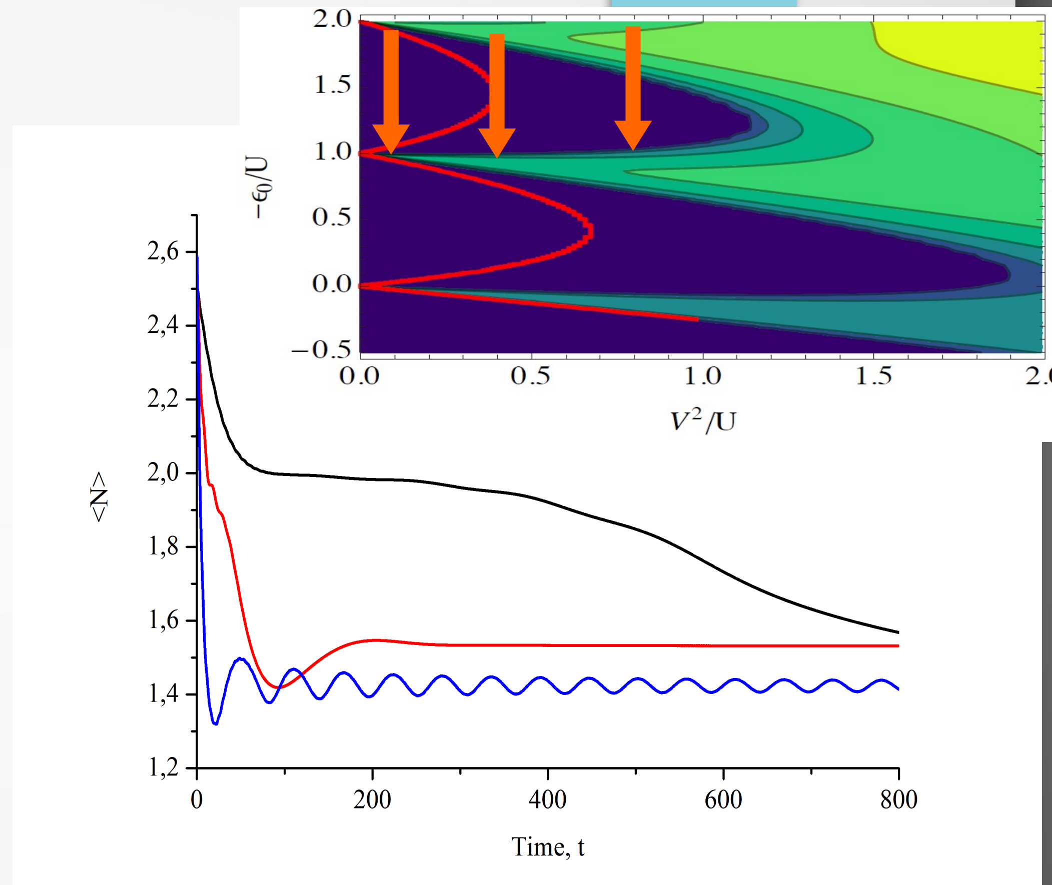




A strong increase of fluctuations while quenching between symmetric phases!

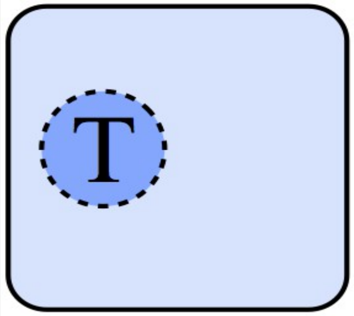
b-SIAM: quench from BEC to BEC phase



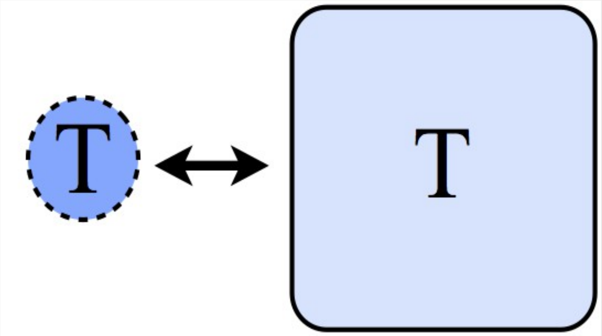


The concept of temperature

Isolated systems



Open systems



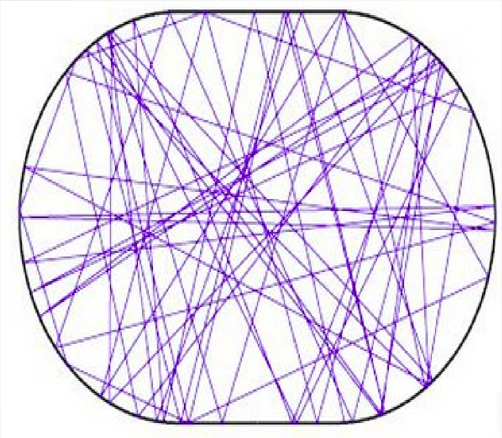
Detailed balance principle

$$r_{m\to n}e^{-E_m/T} = r_{n\to m}e^{-E_n/T}$$

Macroscopic limit

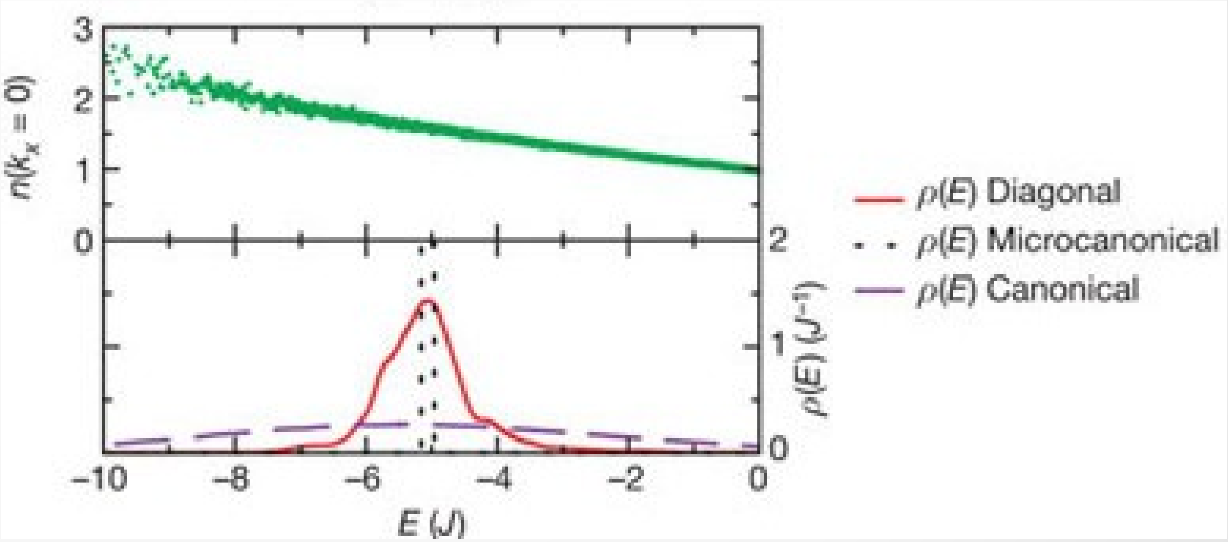
Thermalization of isolated systems

Classical systems:
ergodic hypotesis



All phase space available (trajectory is a microcanonical ensemble)

Quantum systems:
Eigenstate thermalization hypotesys

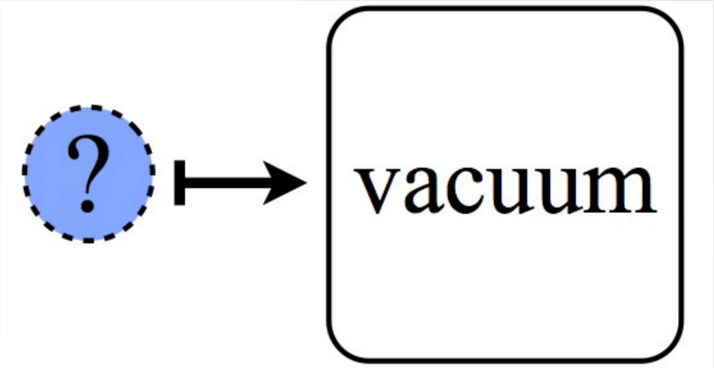


Observables almost do not vary between eigenstates close in energy
(eigenstate is a microcanonical ensemble)

M. Rigol et al. Nature 452, 854 (2008)

Thermalization without detailed balance

Macroscopic limit: averages

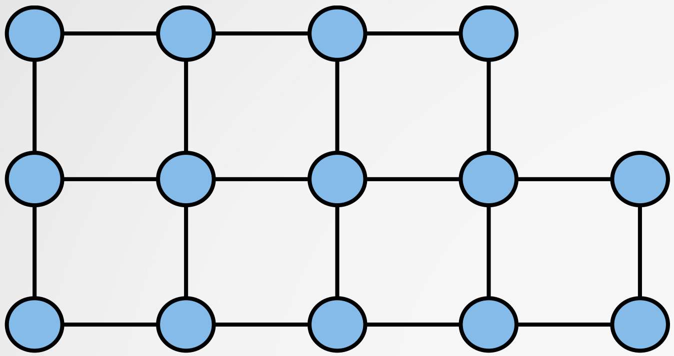


emissive system

Finite-size: Boltzmann (???) distribution

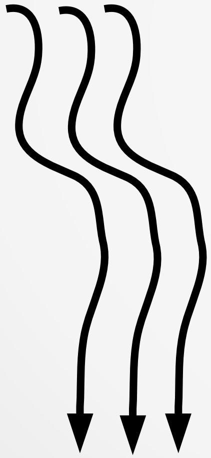
Hard-core bosons on a lattice with emission

System



$$H_S = -\sum_{\langle ij \rangle} h_{ij}(b_i^\dagger b_j + b_j^\dagger b_i)$$

Constraint: each site is occupied no more then once



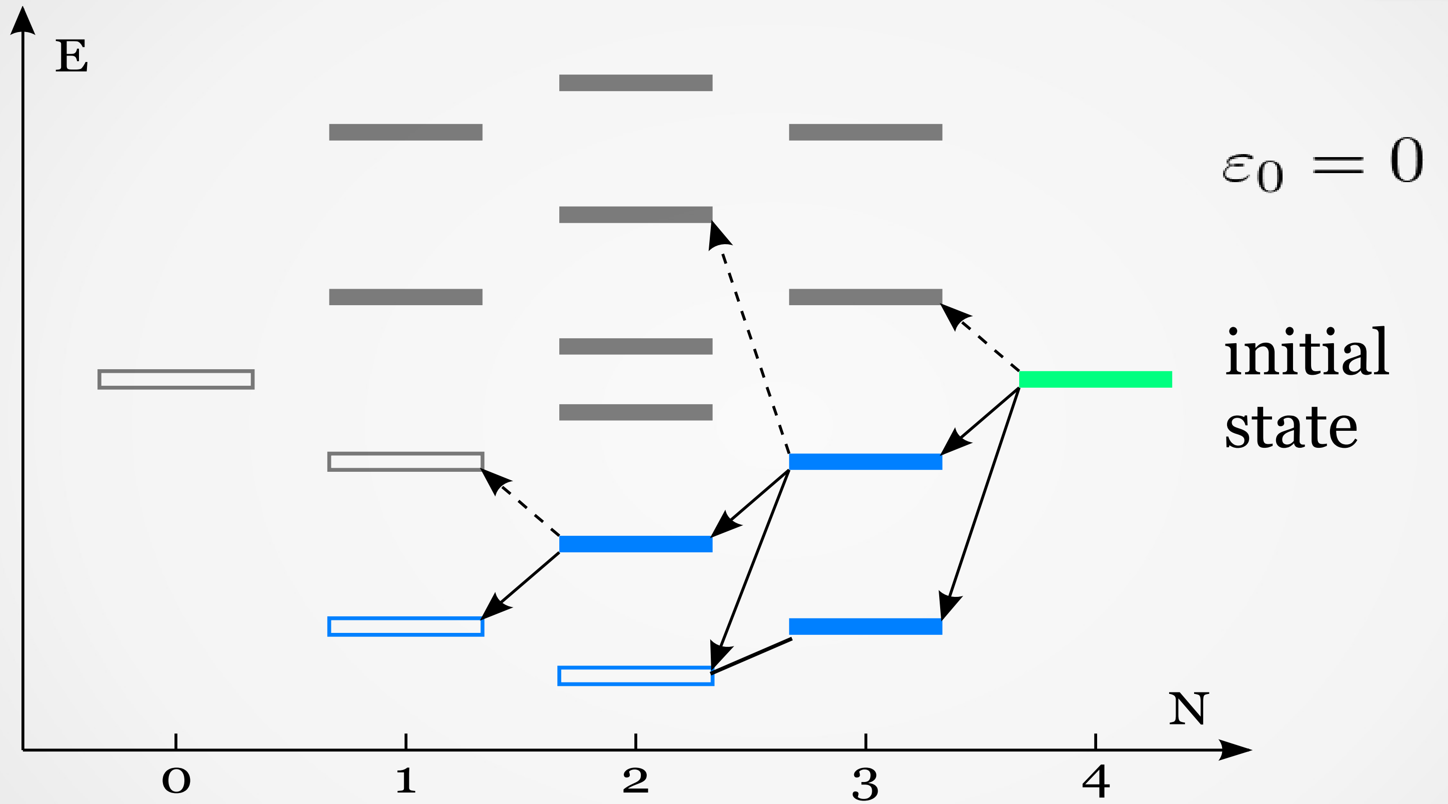
Coupling

$$H_I = \alpha \sum_{ki} (a_k^\dagger b_i + b_i^\dagger a_k)\theta(\varepsilon_k - \varepsilon_0)$$

Reservoir

$$H_R = \sum_k \varepsilon_k a_k^\dagger a_k$$

The evolution of a system at 4-site lattice



Energy barrier results in a number of populated stable states.
We ask how these states are populated.

Coarse-grained master equation

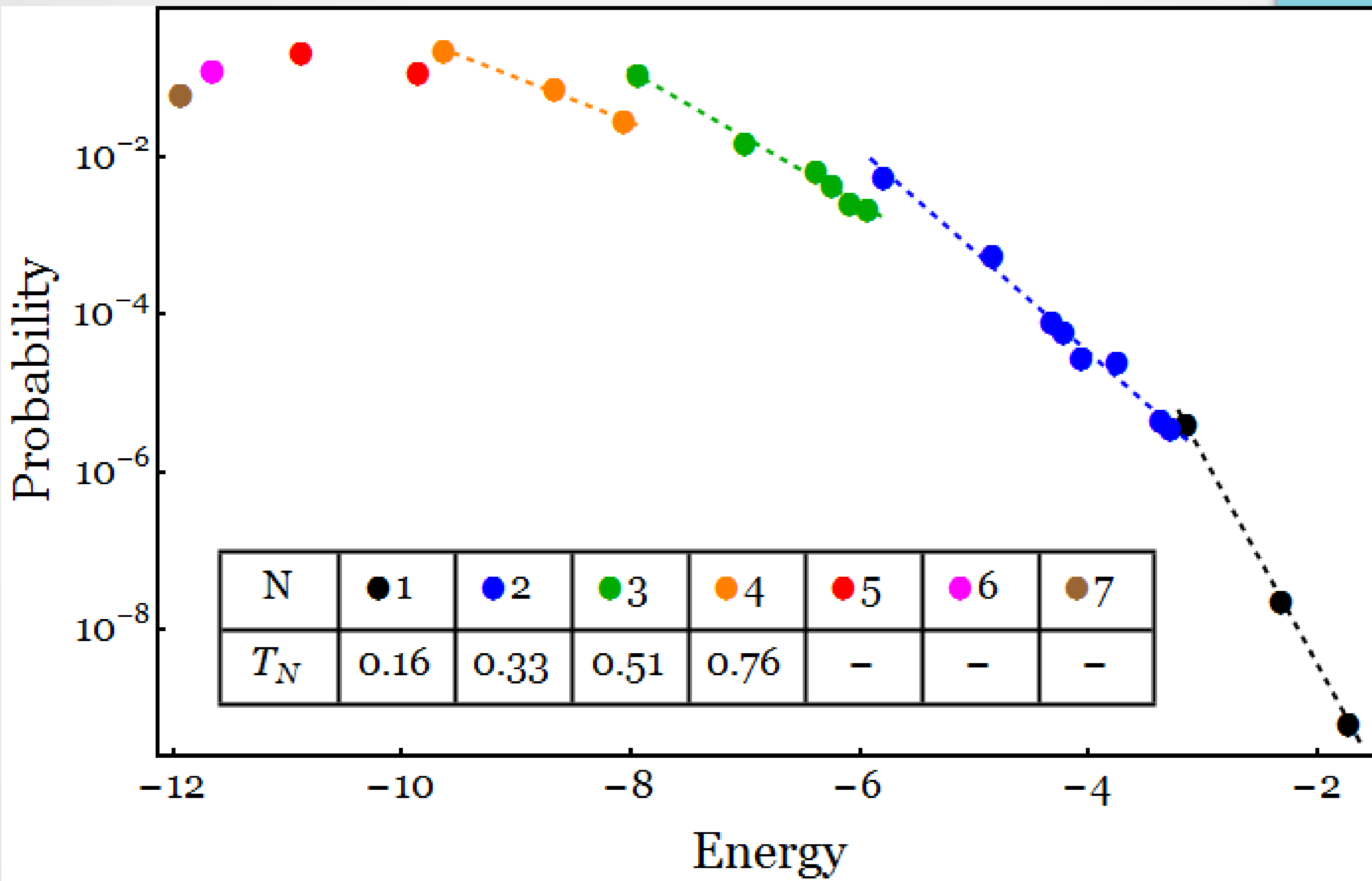
$$\frac{d}{dt}P_n^N = \sum_m R_{nm}^{N+1} P_m^{N+1} - \sum_m R_{mn}^N P_n^N$$

$$\langle N, n|\rho_S|N, n\rangle \equiv P_n^N$$

Fermi's golden rule

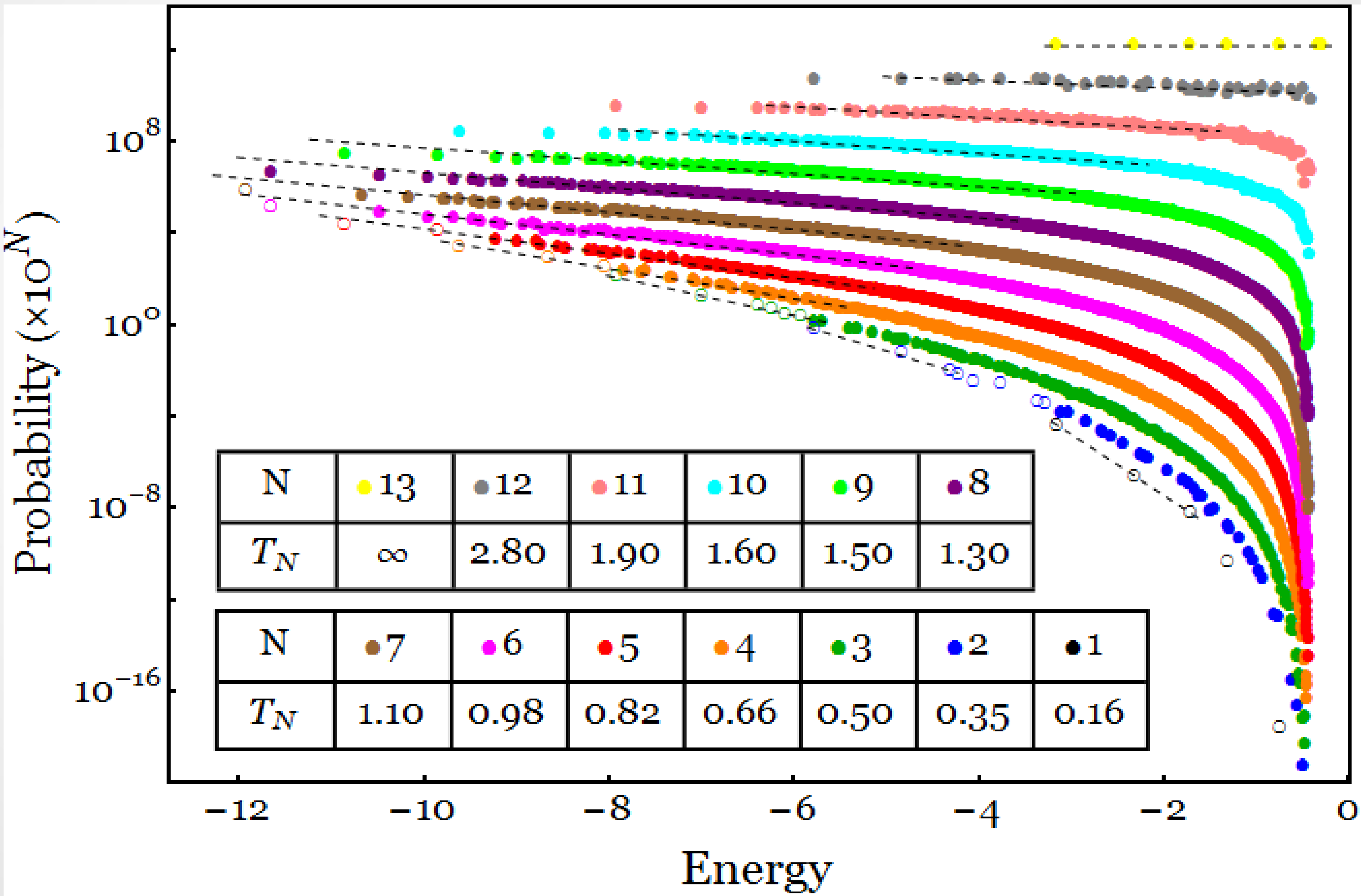
$$R_{mn}^N = 2\pi\Omega_0\alpha^2 \sum_{i=1}^{L} |\langle N-1, m|\hat{b}_i|N, n\rangle|^2$$

Distribution over stable states

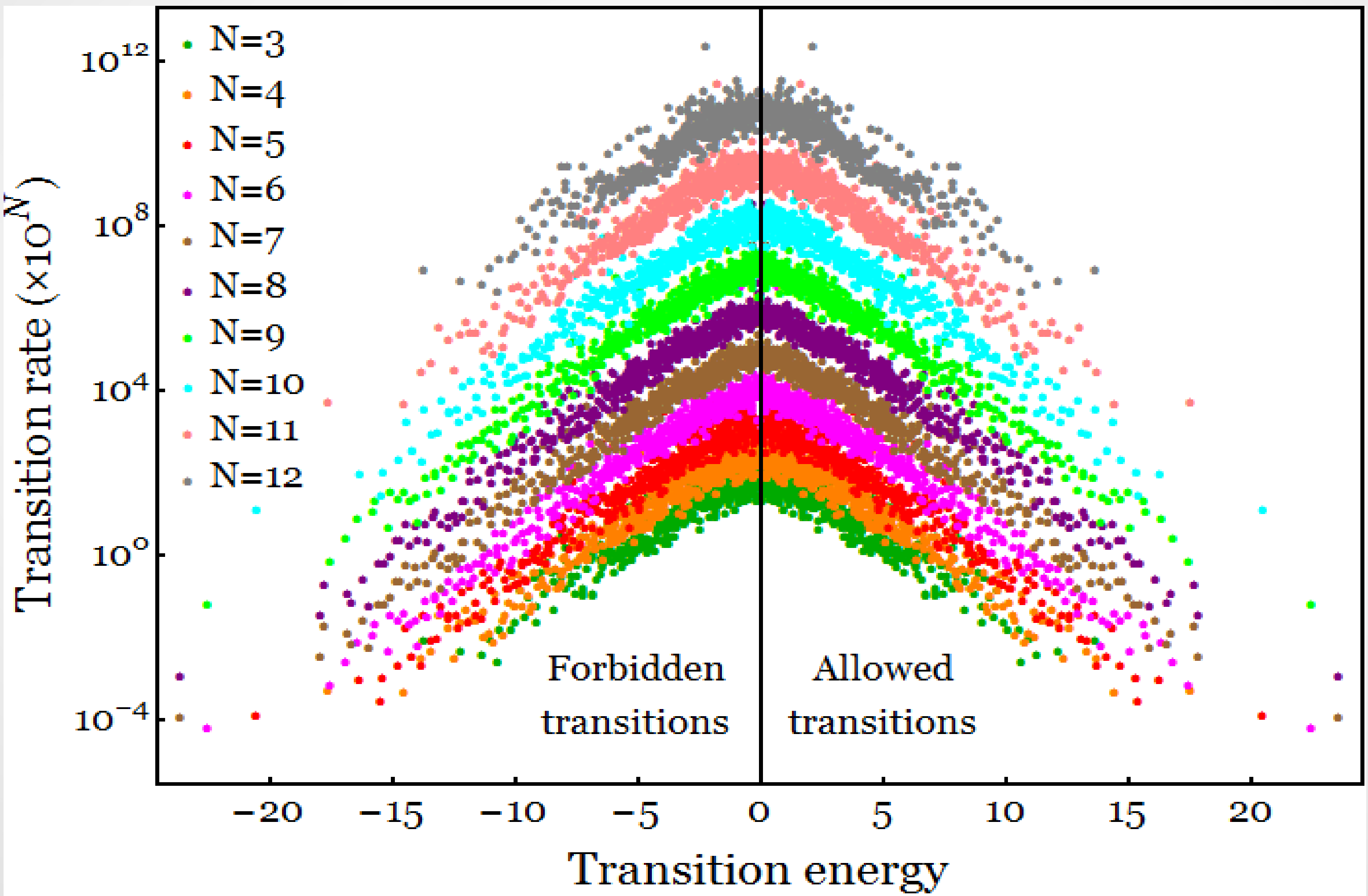


Bolzmann statistics in the sectors with same particle numbers

Distribution over intermediate states

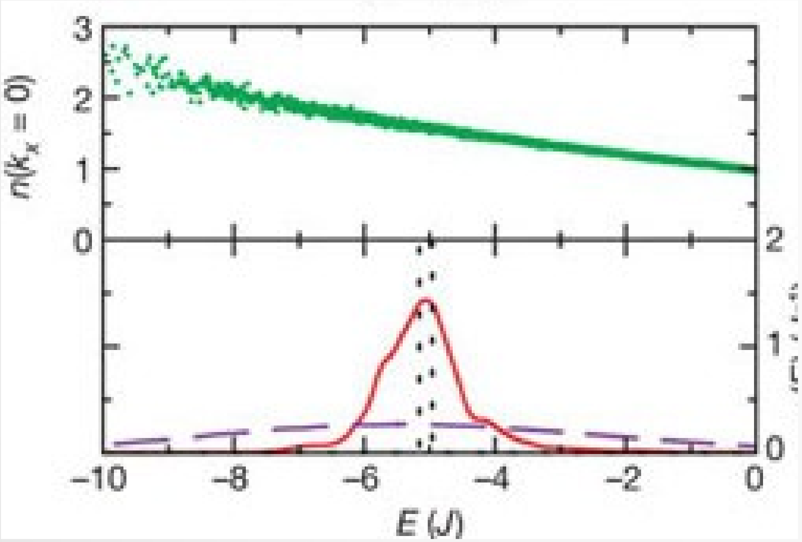


Transition rate vs. transition energy



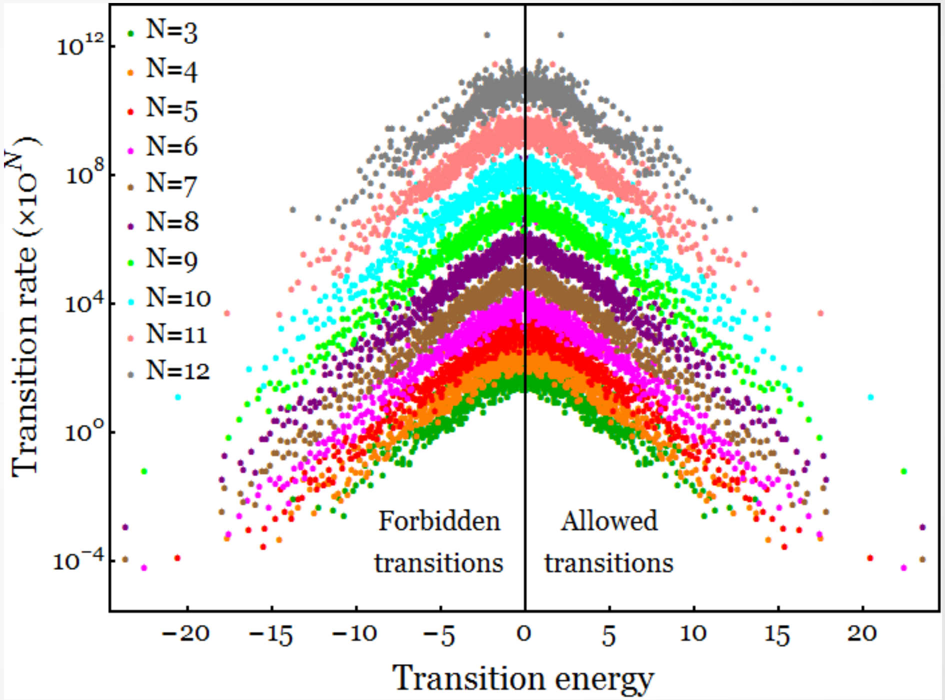
Roots of the "temperature”

Isolated quantum systems:
Eigenstate thermalization hypotesys



Each eigenstate itself forms a microcanonical ensemble

Emissive quantum systems:
This work



Emissing of particles forms a canonical ensemble at each N-particle sector