«Проблемы физики твердого тела и высоких давлений» - Сочи-2015

Теоретическое описание неравновесной динамики фазовых переходов в классических и квантовых конденсированных системах

Михаил Васин

ФТИ УрО РАН,
ИФВД РАН

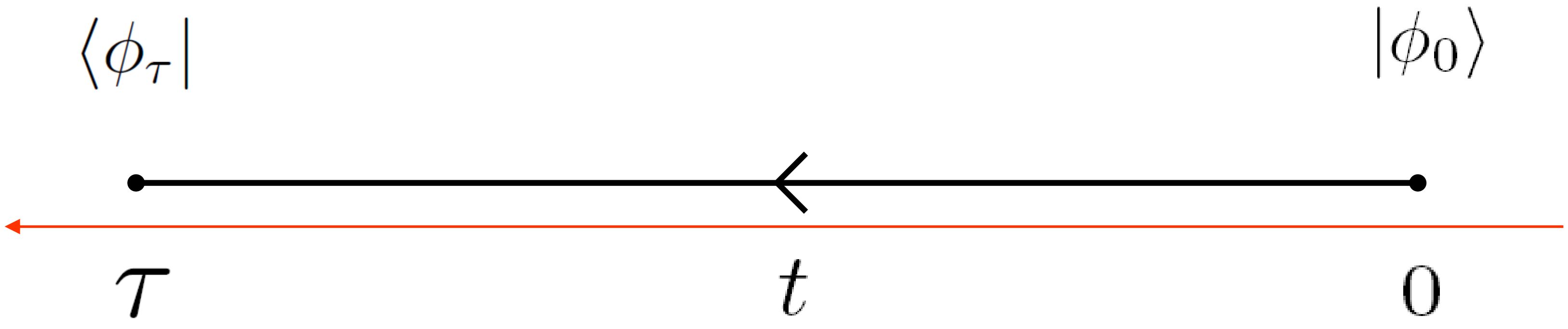
План:

- Келдышевская техника
- Квантово-классический кроссовер вблизи квантовой критической точки
- Описание перехода жидкость-стекло

Интеграл по траекториям

Келдышевская техника

Интеграл по траекториям



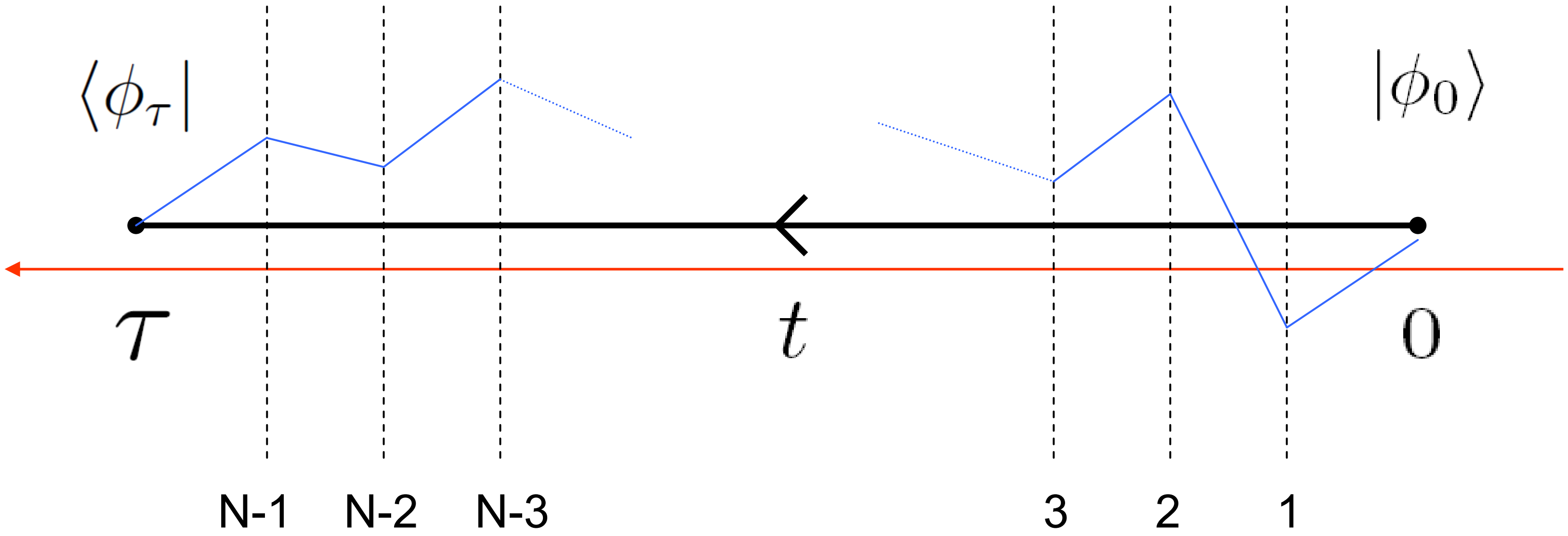
Положим, что $\langle\phi|\phi'\rangle = \delta(\phi - \phi')$

Амплитуда перехода из состояния $|\phi_0\rangle$ **в состояние** $\langle\phi_\tau|$ **.**

$$\langle\phi_\tau|U|\phi_0\rangle = \langle\phi_\tau|e^{-\frac{i}{\hbar}H\tau}|\phi_0\rangle$$

Келдышевская техника

Интеграл по траекториям



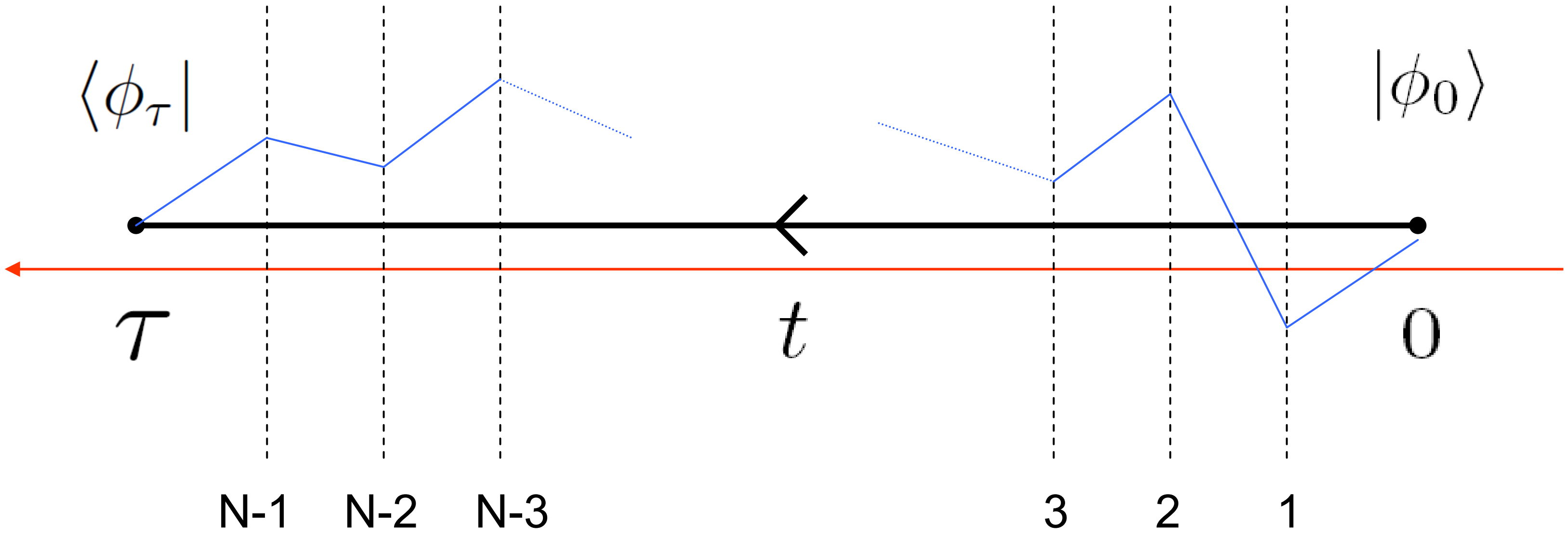
$$\langle\phi_\tau|U|\phi_0\rangle =$$

$$= \iint \langle\phi_\tau|e^{-\frac{i}{\hbar}H_N(\tau - t_{N-1})}|\phi_{N-1}\rangle\langle\phi_{N-1}|e^{-\frac{i}{\hbar}H_{N-1}(t_{N-1}-t_{N-2})}|\phi_{N-2}\rangle\ldots$$

$$\ldots\langle\phi_2|e^{-\frac{i}{\hbar}H_1(t_2-t_1)}|\phi_1\rangle\langle\phi_1|e^{-\frac{i}{\hbar}H_0(t_1-0)}|\phi_0\rangle \prod_{i=1}^{N-1} d\phi_i =$$

Келдышевская техника

Интеграл по траекториям



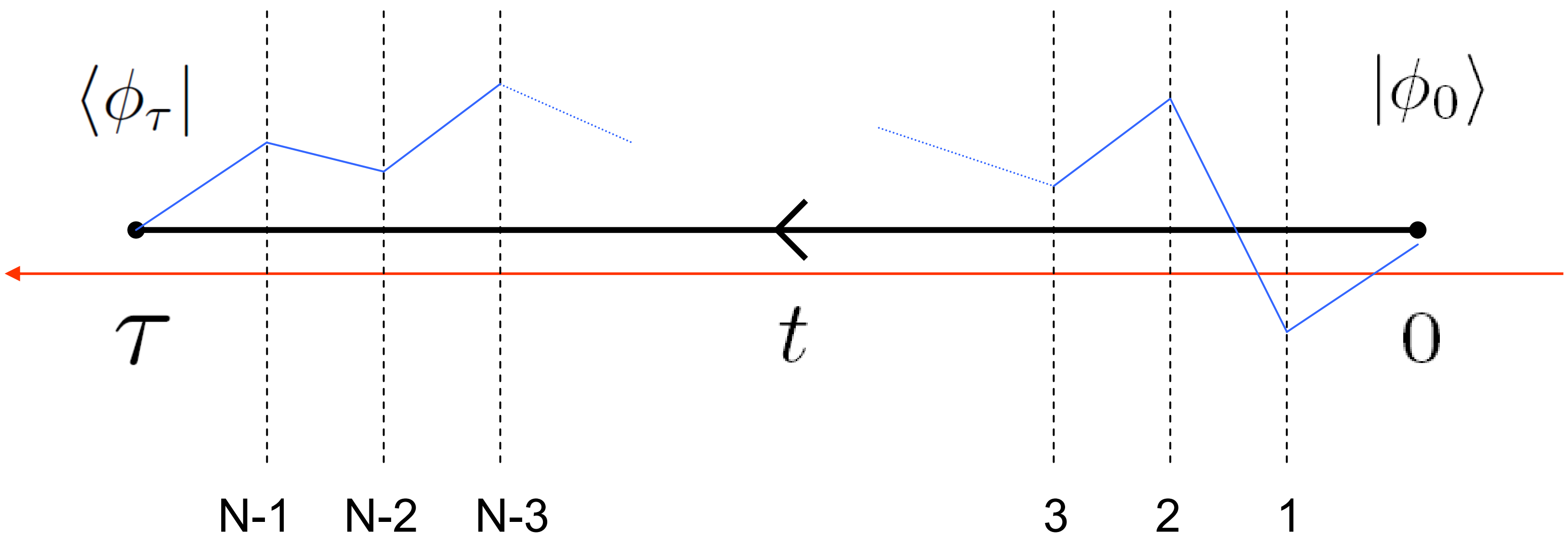
$$\langle\phi_\tau|U|\phi_0\rangle =$$

$$= \iint \langle\phi_\tau| e^{-\frac{i}{\hbar}H_N(\tau - t_{N-1})} e^{-\frac{i}{\hbar}H_{N-1}(t_{N-1}-t_{N-2})} \ldots$$

$$\ldots e^{-\frac{i}{\hbar}H_2(t_2-t_1)} e^{-\frac{i}{\hbar}H_1(t_1-0)} |\phi_0\rangle \prod_{i=1}^{N-1} d\phi_i =$$

Келдышевская техника

Интеграл по траекториям



$$(\tau - t_{N-1}) = (t_{N-1} - t_{N-2}) = \cdots = (t_2 - t_1) = (t_1 - 0) = \Delta t \to dt$$

$$\langle \phi_\tau | U | \phi_0 \rangle = \iint \langle \phi_\tau | e^{-\frac{i}{\hbar} H_\tau \Delta t - \frac{i}{\hbar} \int\limits_0^\tau H(t) dt} | \phi_0 \rangle \mathfrak{D}\phi$$

$$\mathfrak{D}\phi \to \prod_{i=1}^{N-1} d\phi_i$$

Келдышевская техника

Статистическая физика

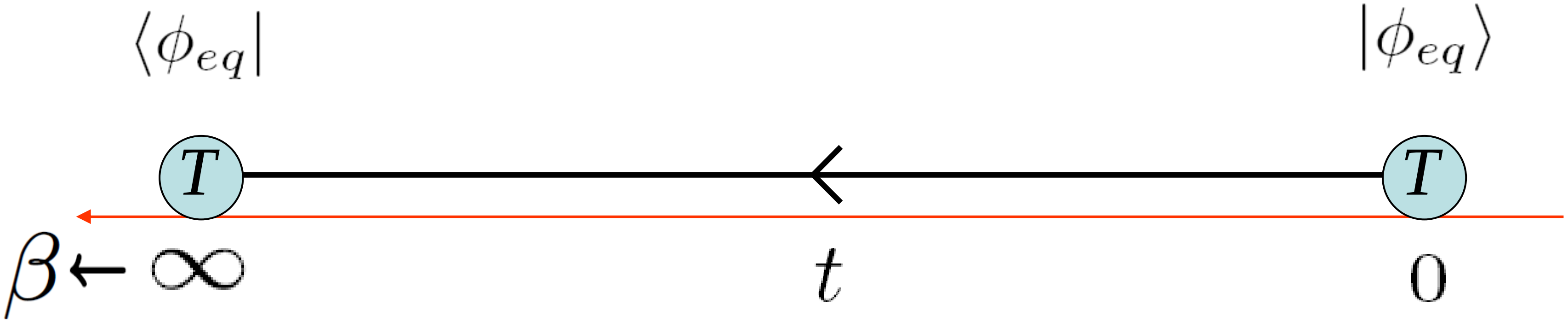
Келдышевская техника

Аналогия между классической статистической механикой и квантовой механикой:

В квантовом случае обратная температура играет роль мнимого времени.

Келдышевская техника

Описание равновесной системы



$$\langle \phi_{eq}|\tilde{U}|\phi_{eq}\rangle = \text{const}$$

$$1 = \langle \phi_{eq}|\tilde{U}|\phi_{eq}\rangle = N\int \mathfrak{D}\phi \exp\left[i\int_0^{\infty} \phi G^{-1}\phi d\tau\right] =$$

$$= N\int \mathfrak{D}\phi \exp\left[-\int_0^{\beta} \phi G^{-1}\phi dt\right] \to N\int \mathfrak{D}\phi \exp\left[-\beta H(\phi)\right]$$

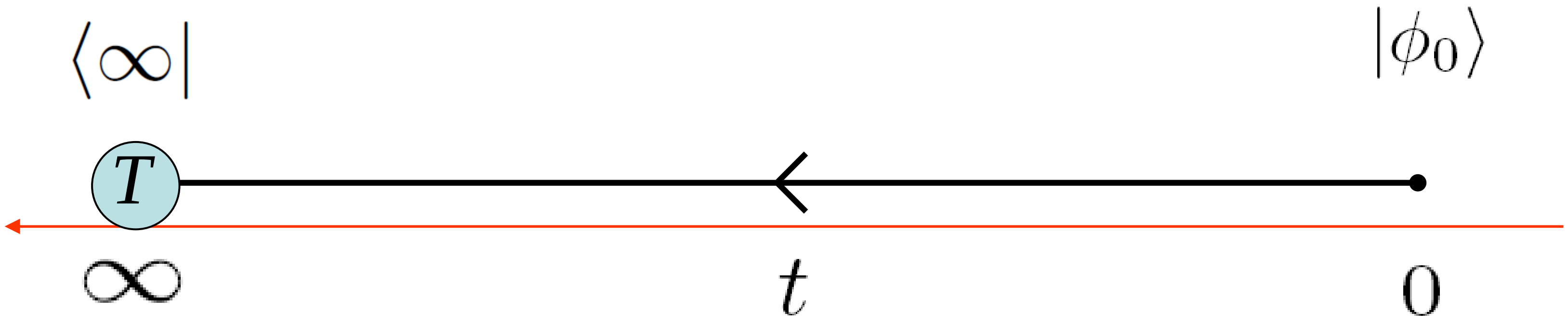
статика

$$\tau = it$$

Неравновесная статистическая механика

Келдышевская техника

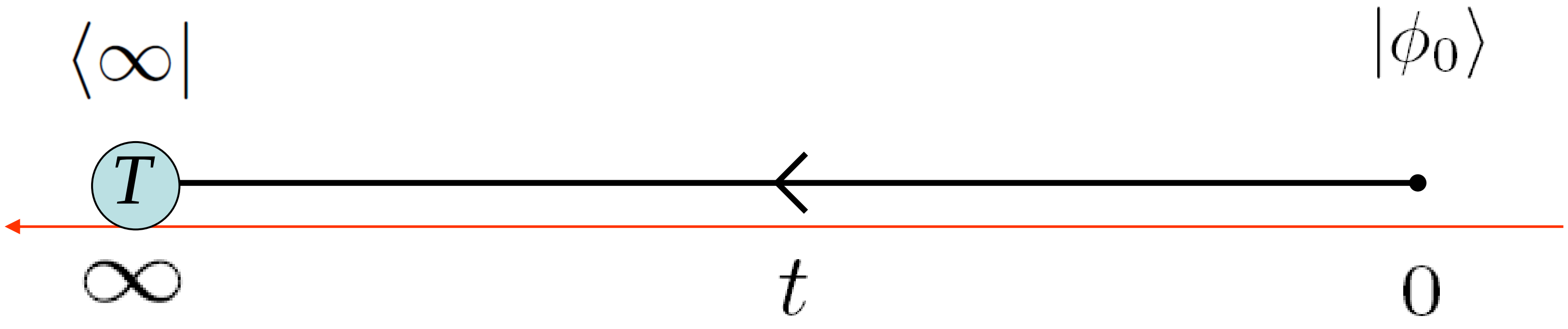
Описание неравновесной бозе-системы



$$\langle \infty | U | \phi_0 \rangle$$

Келдышевская техника

Описание неравновесной бозе-системы



$$\langle\infty|U|\phi_0\rangle$$

$$U = e^{-\frac{i}{\hbar}\int(\gamma\phi E\phi+\phi G^{-1}\phi)d\omega}$$

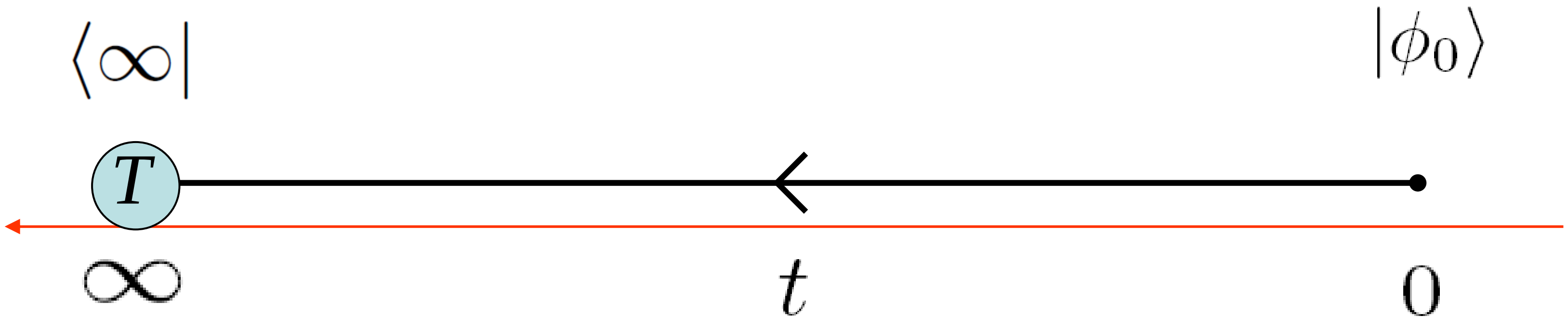
$$G^{-1} = \varepsilon - i\gamma\omega$$

Система представляет собой ансамбль квантовых гармонических осцилляторов, взаимодействующей с тепловым резервуаром

$$E = \hbar\omega(n(\omega)+1/2) = \frac{1}{2}\hbar\omega\coth(\omega/T)$$

Келдышевская техника

Описание неравновесной бозе-системы

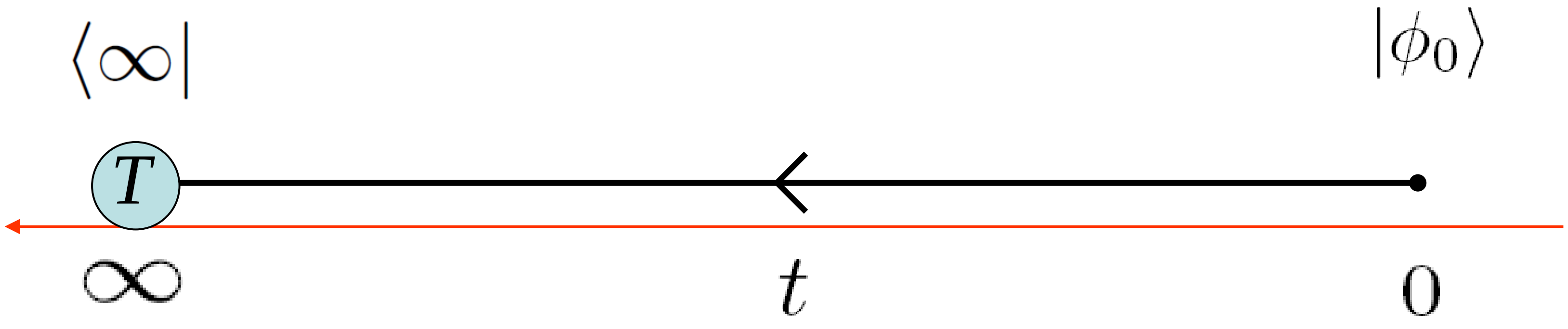


$$\langle \infty | U | \phi_0 \rangle$$

$$U = \int \mathfrak{D}\phi \exp\left[\frac{i}{\hbar} \int_0^\infty \left\{ \phi G^{-1} \phi + \gamma \hbar \omega \coth(\omega / T) \phi \phi \right\} d\omega \right] .$$

Келдышевская техника

Описание неравновесной бозе-системы



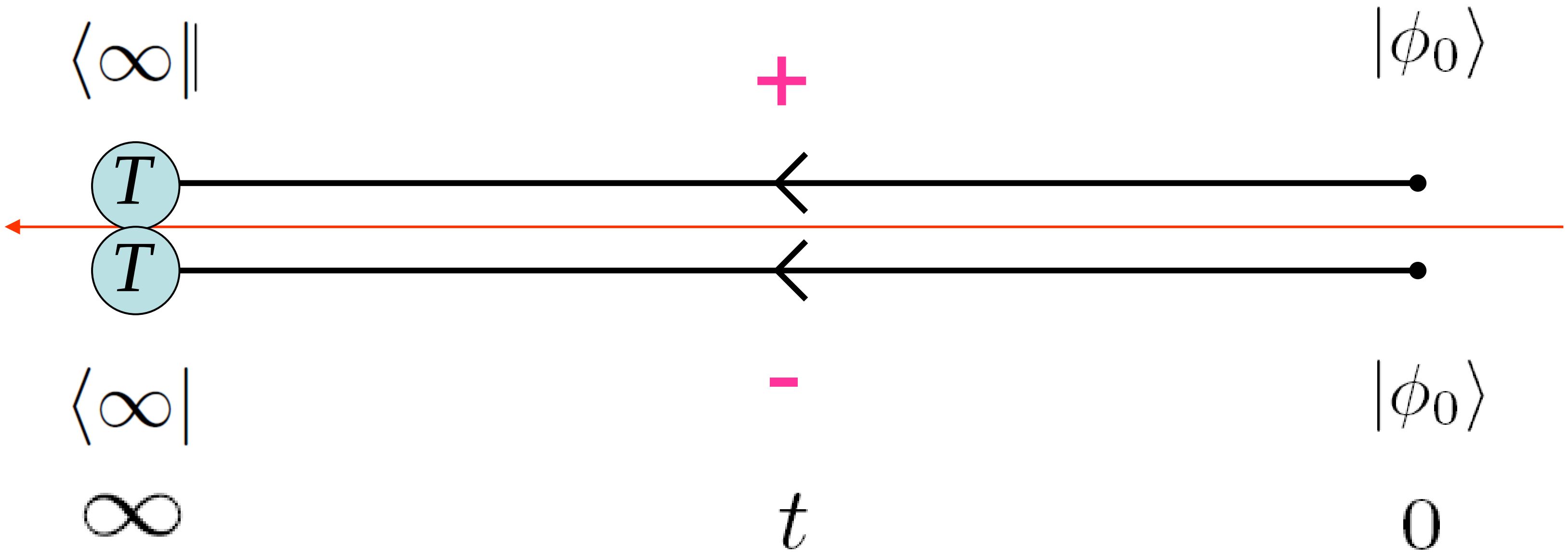
$$\langle \infty | U | \phi_0 \rangle = f(\phi_0) \quad ?$$

$$U = \int \mathfrak{D}\phi \exp\left[\frac{i}{\hbar}\int_0^{\infty}\left\{\phi G^{-1}\phi + \gamma\hbar\omega \coth(\omega/T)\phi\phi\right\} d\omega\right].$$

Операция усреднения не определена, поскольку зависит от начальных условий т.е. от $|\phi_0\rangle$ **!**

Келдышевская техника

Описание неравновесной бозе-системы

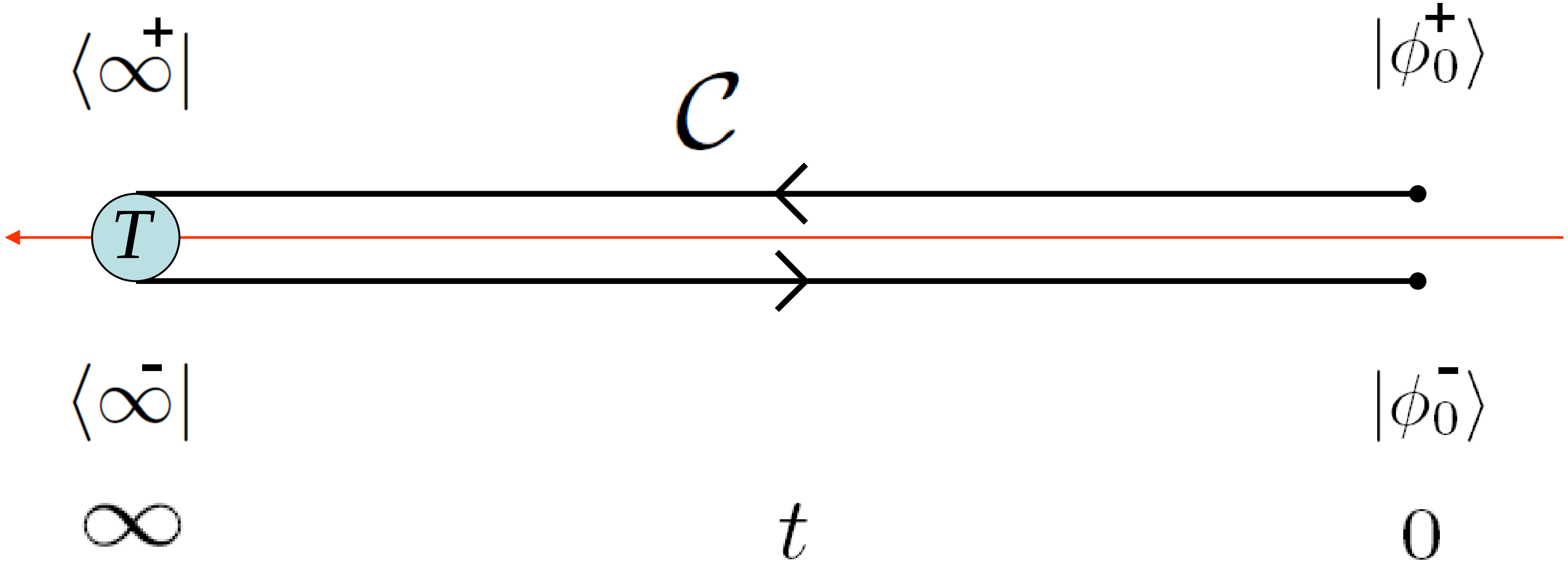


$$U_1 = \int \mathfrak{D}\phi^+ \exp\left[\frac{i}{\hbar}\int\limits_0^\infty \left\{\phi^+ G^{-1}\phi^+ + \gamma\hbar\omega\coth(\omega/T)\phi^+\phi^+\right\} d\omega\right].$$

$$U_2 = \int \mathfrak{D}\phi^- \exp\left[\frac{i}{\hbar}\int\limits_0^\infty \left\{\phi^- G^{-1}\phi^- + \gamma\hbar\omega\coth(\omega/T)\phi^-\phi^-\right\} d\omega\right].$$

Келдышевская техника

Описание неравновесной бозе-системы



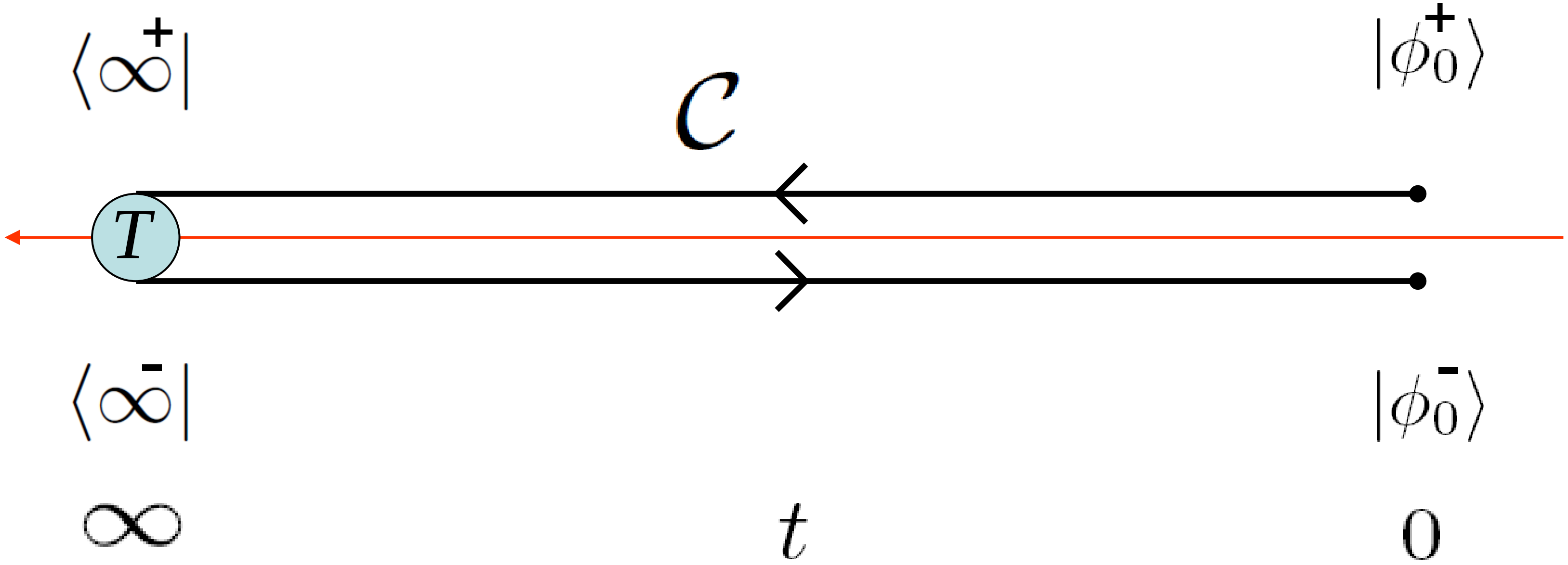
$$U_{\mathcal{C}} \equiv 1$$

$$\langle \phi_0^{-} | \phi_0^{+} \rangle = 1$$

Schwinger

Келдышевская техника

Описание неравновесной бозе-системы



$$U_{\mathcal{C}} = \int \mathfrak{D}\phi^{+}\mathfrak{D}\phi^{-} \exp\left[\frac{i}{\hbar}\int\limits_{0}^{\infty}\left\{\phi^{+}G^{-1}\phi^{+} - \phi^{-}G_{-}^{-1}\phi^{-}\right.\right.$$

$$\left.\left.-\gamma\hbar\omega\coth(\omega/T)(\phi^{+}\phi^{+} + \phi^{-}\phi^{-} - \phi^{+}\phi^{-} - \phi^{-}\phi^{+})\right\} d\omega\right].$$

Келдышевская техника

$$U_{\mathcal{C}} \equiv 1$$

Описание неравновесной бозе-системы

+

Келдышевский поворот

$$\phi^{cl} = \sqrt{1/2}(\phi^+ + \phi^-), \phi^q = \sqrt{1/2}(\phi^+ - \phi^-)$$

$$\phi^+ = \sqrt{1/2}(\phi^{cl} + \phi^q),\ \phi^- = \sqrt{1/2}(\phi^{cl} - \phi^q)$$

$$U_C = \int \mathfrak{D}\phi^+ \mathfrak{D}\phi^- \exp\left[\frac{i}{\hbar}\int_0^\infty \left\{\phi^+ G^{-1}\phi^+ - \phi^- G^{-1}\phi^-\right.\right.$$

$$\left.\left. -\gamma\hbar\omega\coth(\omega/T)(\phi^+\phi^+ + \phi^-\phi^- - \phi^+\phi^- - \phi^-\phi^+)\right\} d\omega\right].$$

Келдышевская техника

Описание неравновесной бозе-системы

+

Келдышевский поворот

$$\phi^{cl} = \sqrt{1/2}(\phi^+ + \phi^-), \phi^q = \sqrt{1/2}(\phi^+ - \phi^-)$$

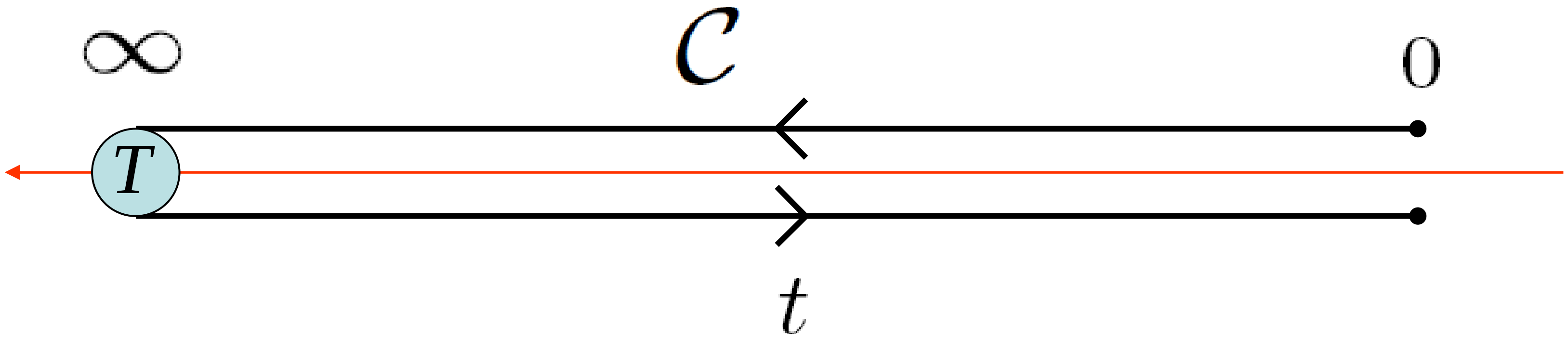
$$\phi^+ = \sqrt{1/2}(\phi^{cl} + \phi^q), \ \phi^- = \sqrt{1/2}(\phi^{cl} - \phi^q)$$

$$U_C = \int \mathfrak{D}\phi^{cl}\mathfrak{D}\phi^q \exp\left[\frac{i}{\hbar}\int_0^\infty \phi^q(\varepsilon + i\gamma\omega)\phi^{cl} d\omega + \right.$$

$$\left. + \frac{i}{\hbar}\int_0^\infty \phi^{cl}(\varepsilon - i\gamma\omega)\phi^q d\omega - \frac{i}{\hbar}\int_0^\infty 2\gamma\omega \coth(\beta\omega/2)\phi^q\phi^q d\omega\right]$$

Келдышевская техника

Описание неравновесной бозе-системы



$$Z = N \int \mathfrak{D}\phi^{cl}\mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl}, \phi^{q})\right]$$

(сделан виковский поворот)

$$S(\phi^{cl}, \phi^{q}) = \int d\omega dk \left(\bar{\phi}\hat{G}^{-1}\bar{\phi} + U(\phi_{cl} + \phi_{q}) - U(\phi_{cl} - \phi_{q})\right)$$

$$\hat{G}^{-1} = \begin{array}{cc} & R \\ \multicolumn{2}{c}{\left[\begin{array}{c|c} 0 & \omega^2 + \varepsilon_k + i\gamma\omega \\ \hline \omega^2 + \varepsilon_k - i\gamma\omega & 2\gamma\omega \coth(\omega/T) \end{array}\right]} \\ A & K \end{array}, \quad \bar{\phi} = \{\phi^{q}, \phi^{cl}\}$$

Литература:

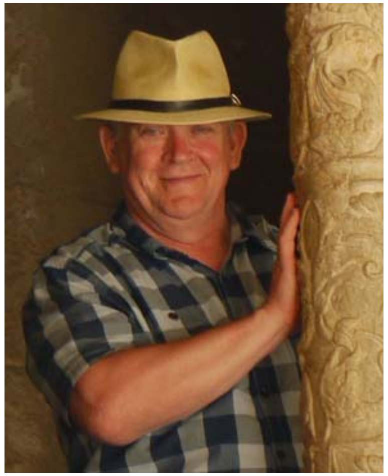
Many–body theory of non–equilibrium systems

Alex Kamenev

Department of Physics, University of Minnesota,
Minneapolis, MN 55455, USA

Квантово-классический кроссовер вблизи квантовой критической точки

Винокур В.М.



Рыжов В.Н.

Васин М.Г.[1,2], Рыжов В.Н.[2], Винокур В.М.[3]

1 – ФТИ УрО РАН, Ижевск, Россия;
2 – ИФВД РАН, Ижевск, Россия
3 - Materials Science Division, Argonne National Laboratory, Argonne, USA

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Stishov, S. M., Petrova, A. E., Gavrilkin, S. Yu., Klinkova, L. A. Quantum degradation of a second-order phase transition. *Phys. Rev.* B **91**, 144416 (2015).

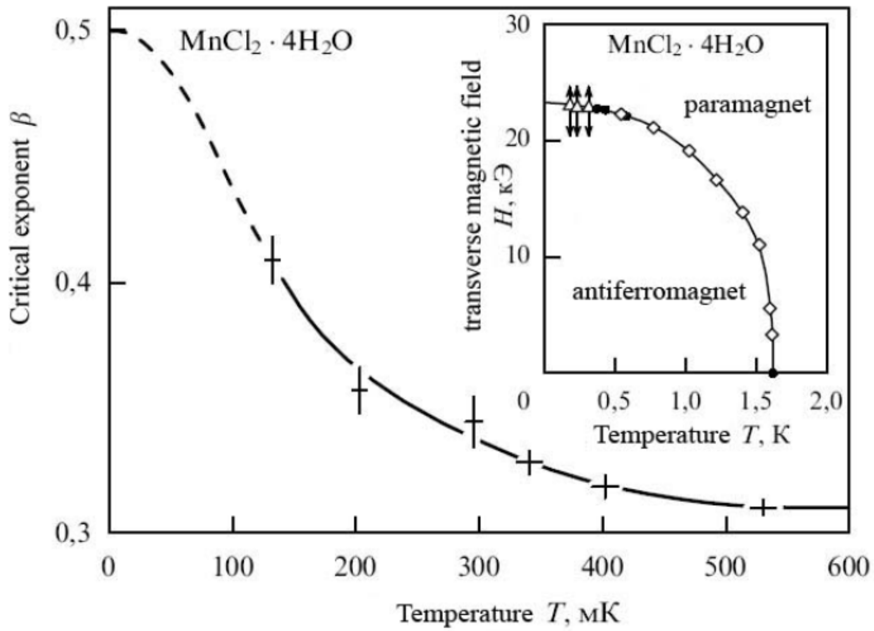

Figure 3: The dependence of critical exponent β determining the order parameter behavior, on the temperature of the antiferromagnet $MnCl_2{\cdot}4H_2O$[24].

d → d+1

P. Pfeuty: *Ann. Phys. (N. Y.)*, **57**, 79 (1970).
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A. P. Young: *J. Phys. C*, **8**, L309 (1975).
J. A. Hertz: *Proceedings of the Conference on Magnetism* (San Francisco, Cal., 1978).
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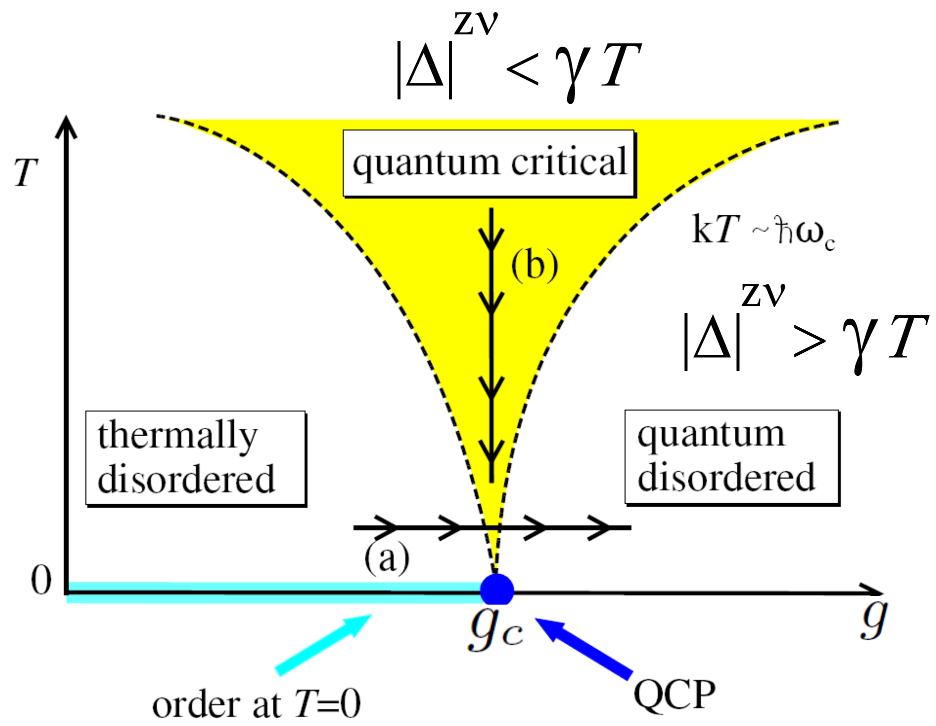
QUANTUM PHASE TRANSITIONS

SUBIR SACHDEV
Professor of Physics,
Yale University,
New Haven, CT 06520-8120, USA

Institute of Physics Publishing
Rep. Prog. Phys. **66** (2003) 2069–2110

Quantum phase transitions

Matthias Vojta

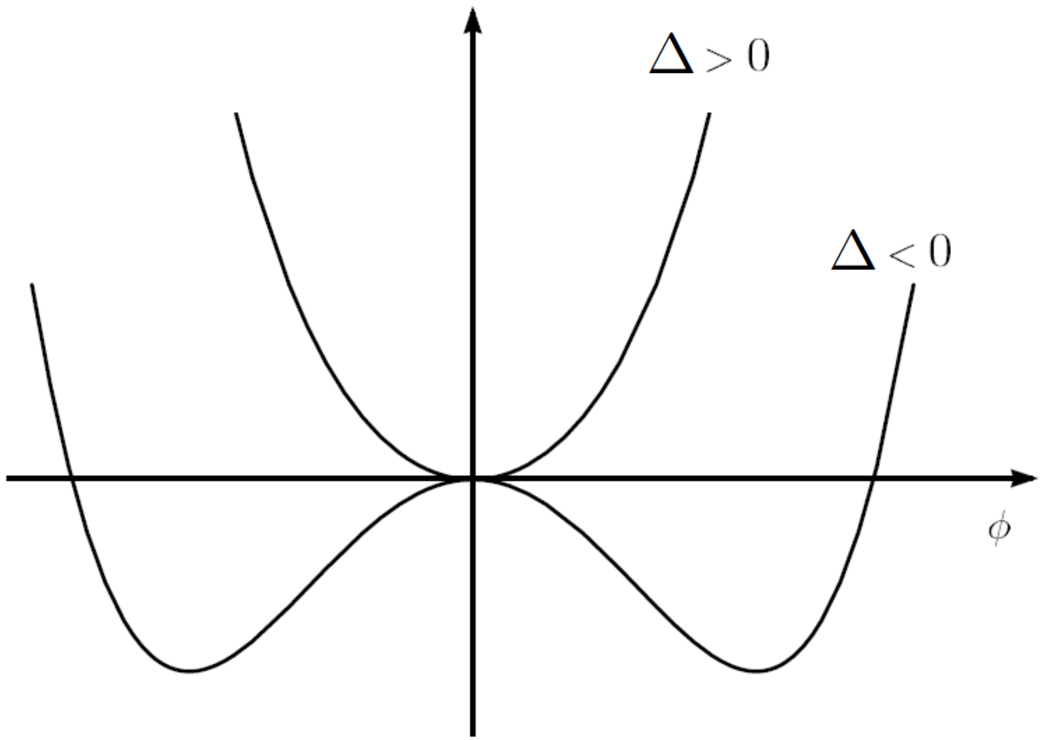


Необходимо рассмотреть критическую динамику вблизи ККТ

Фазовый переход в ϕ^4 - модели

ϕ^4 **-модель** $U \approx \Delta(g)\phi^2 + v\phi^4$

$$\Delta(g) = (g - g_c)$$



Статика
(классическая статистическая механика)

$$Z = N \int \mathfrak{D}\phi \exp\left[-S(\phi)\right]$$

$$S(\phi) = \frac{1}{T} \int dk \left(\phi G^{-1} \phi + U(\phi)\right)$$

$$G^{-1} = \varepsilon_k = k^2 + \Delta$$

$$\Delta(g) = (g - g_c)$$

N – **нормировочный множитель,**
g – **контролирующий параметр**

MV1

A. Kamenev and A. Levchenko, Advances in Physics 58, 197 (2009);

Динамика

(квантовая/классическая статистическая механика бозе-системы в келдышевской технике)

$$Z = N \int \mathfrak{D}\phi^{cl}\mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl},\phi^{q})\right]$$

(сделан виковский поворот)

$$S(\phi^{cl},\phi^{q}) = \int d\omega dk \left(\bar{\phi}\hat{G}^{-1}\bar{\phi} + U(\phi_{cl}+\phi_{q}) - U(\phi_{cl}-\phi_{q})\right)$$

$$\hat{G}^{-1} = \left[\begin{array}{c|c} 0 & \overset{R}{\omega^2+\varepsilon_k+i\gamma\omega} \\ \hline \underset{A}{\omega^2+\varepsilon_k-i\gamma\omega} & \underset{K}{\underline{2\gamma\omega\coth(\omega/T)}} \end{array}\right], \quad \bar{\phi} = \{\phi^{q},\, \phi^{cl}\}$$

$$G^{-1\,K} = i\coth(\omega/T)\left[G^{-1\,A} - G^{-1\,R}\right]$$ - ФДТ

Слайд 29

MV1 От статической задачи динамическая отличается удвоенным количеством полей. Существует несколько техник описания неравновесной динамики. В данном случае приведена формулировка в рамках Келдышевской техники, в которой при определении пары полей выполняется т.н. КЕЛДЫШЕВСКИЙ поворот. Это очень удобное представление.
- Флуктуации = статансамль (в данном случае рассматривается система с бозе-статистикой)
- Флкутуации = диссипация (ФДТ)

Mikhail Vasin; 12.02.2014

V1

Динамика

(квантовая/классическая статистическая механика
бозе-системы в келдышевской технике)

$$Z = N \int \mathfrak{D}\phi^{cl}\mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl},\phi^{q})\right]$$

$$S(\phi^{cl},\phi^{q}) = \int d\omega dk \left(\bar{\phi}\hat{G}^{-1}\bar{\phi} + U(\phi_{cl}+\phi_q) - U(\phi_{cl}-\phi_q)\right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \omega^2+\varepsilon_k+i\gamma\omega \\ \omega^2+\varepsilon_k-i\gamma\omega & 2\gamma\omega\coth(\omega/T) \end{bmatrix}, \quad \bar{\phi} = \{\phi^q,\, \phi^{cl}\}$$

This function determines the crossover between
quantum and classical limits

Слайд 30

V1 VASIN; 16.02.2014

Динамика
(классический предел)

$$T \gg \omega$$

$\boxed{\hbar = 1 \text{ and } k_B = 1}$

$$Z = N \int \mathfrak{D}\phi^{cl} \mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl}, \phi^{q})\right]$$

$$S(\phi^{cl}, \phi^{q}) = \int d\omega dk \left(\bar{\phi}\hat{G}^{-1}\bar{\phi} + U(\phi_{cl} + \phi_{q}) - U(\phi_{cl} - \phi_{q})\right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \omega^2 + \varepsilon_k + i\gamma\omega \\ \omega^2 + \varepsilon_k - i\gamma\omega & 2\gamma T \end{bmatrix}, \quad \bar{\phi} = \{\phi^{q}, \phi^{cl}\}$$

Критическая динамика
(классический предел)

$$\underline{T \gg} \omega$$

$$Z = N \int \mathfrak{D}\phi^{cl}\mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl},\phi^{q})\right]$$

$$S(\phi^{cl},\phi^{q}) = \int d\omega dk \left(\bar{\phi}\hat{G}^{-1}\bar{\phi} + U(\phi_{cl}+\phi_q) - U(\phi_{cl}-\phi_q)\right)$$

In the critical (fluctuation) region

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \cancel{\omega^2} + \varepsilon_k + i\gamma\omega \\ \cancel{\omega^2} + \varepsilon_k - i\gamma\omega & \underline{2\gamma T} \end{bmatrix}, \quad \bar{\phi} = \{\phi^q,\, \phi^{cl}\}$$

dispersion

$$\omega \sim k^2 \quad \Rightarrow \quad z = 2 \quad \Rightarrow \quad D = d + 2$$

Динамика

(квантовый предел)

$$T \ll \omega$$

$$Z = N \int \mathfrak{D}\phi^{cl} \mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl}, \phi^{q})\right]$$

$$S(\phi^{cl}, \phi^{q}) = \int d\omega dk \left(\bar{\phi} \hat{G}^{-1} \bar{\phi} + U(\phi_{cl} + \phi_{q}) - U(\phi_{cl} - \phi_{q}) \right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \omega^2 + \varepsilon_k + i\gamma\omega \\ \omega^2 + \varepsilon_k - i\gamma\omega & 2\gamma|\omega| \end{bmatrix}, \quad \bar{\phi} = \{\phi^{q}, \phi^{cl}\}$$

Критическая динамика

(квантовый предел)

$$T \ll \omega$$

$$Z = N \int \mathfrak{D}\phi^{cl} \mathfrak{D}\phi^q \exp\left[-S(\phi^{cl}, \phi^q)\right]$$

$$S(\phi^{cl}, \phi^q) = \int d\omega dk \left(\bar{\phi} \hat{G}^{-1} \bar{\phi} + U(\phi_{cl} + \phi_q) - U(\phi_{cl} - \phi_q) \right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \cancel{\omega^2} + \varepsilon_k + i\gamma\omega \\ \cancel{\omega^2} + \varepsilon_k - i\gamma\omega & 2\gamma|\omega| \end{bmatrix}, \quad \bar{\phi} = \{\phi^q, \phi^{cl}\}$$

dispersion

$$\omega \sim k^2 \quad \Rightarrow \quad z = 2$$

Динамика
(недиссипативный квантовый предел)

$$\underline{\gamma \to 0} \qquad \omega \ll \tau_{rel}^{-1} \qquad T \ll \tau_{rel}^{-1}$$

$$Z = N \int \mathfrak{D}\phi^{cl} \mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl}, \phi^{q})\right]$$

$$S(\phi^{cl}, \phi^{q}) = \int d\omega dk \left(\bar{\phi} \hat{G}^{-1} \bar{\phi} + U(\phi_{cl} + \phi_q) - U(\phi_{cl} - \phi_q) \right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \omega^2 + \varepsilon_k + i\gamma\omega \\ \omega^2 + \varepsilon_k - i\gamma\omega & 2\gamma\omega \coth(\omega/T) \end{bmatrix}, \quad \bar{\phi} = \{\phi^q, \, \phi^{cl}\}$$

Динамика
(недиссипативный квантовый предел)

$$\underline{\gamma \to 0} \qquad \omega \ll \tau_{rel}^{-1} \qquad T \ll \tau_{rel}^{-1}$$

$$Z = N \int \mathfrak{D}\phi^{cl} \mathfrak{D}\phi^{q} \exp\left[-S(\phi^{cl}, \phi^{q})\right]$$

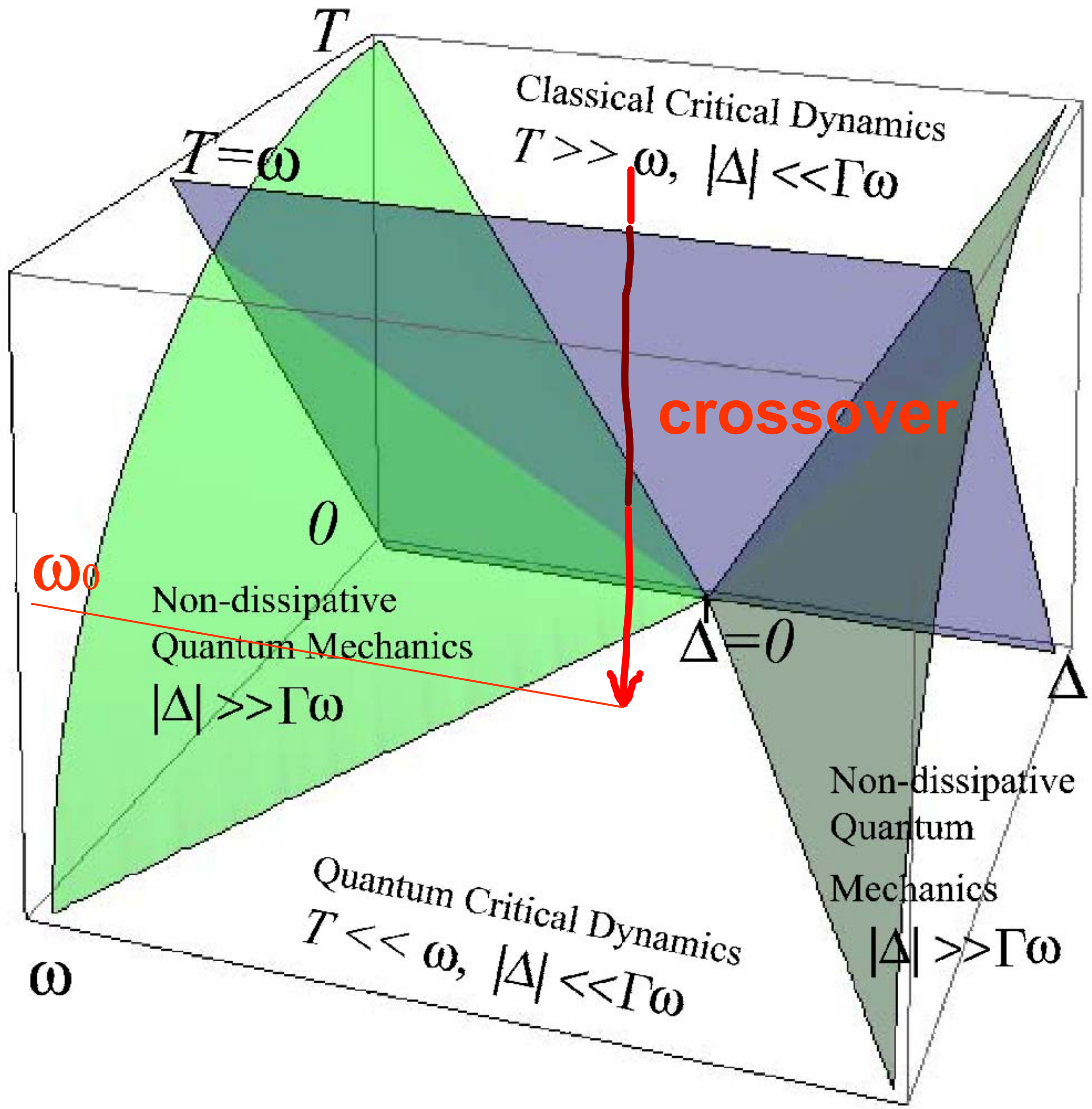
$$S(\phi^{cl}, \phi^{q}) = \int d\omega dk \left(\bar{\phi} \hat{G}^{-1} \bar{\phi} + U(\phi_{cl} + \phi_q) - U(\phi_{cl} - \phi_q) \right)$$

$$\hat{G}^{-1} = \begin{bmatrix} 0 & \omega^2 + \varepsilon_k + \overbrace{\cancel{i\gamma\omega}}^{=0} \\ \omega^2 + \varepsilon_k - \underbrace{\cancel{i\gamma\omega}}_{=0} & \underbrace{\cancel{2\gamma\omega \coth(\omega/T)}}_{=0} \end{bmatrix}, \quad \bar{\phi} = \{\phi^{q}, \phi^{cl}\}$$

дисперсионное соотношение

$$\omega \sim k \quad \Rightarrow \quad z = 1 \quad \Rightarrow \quad D = d + 1$$

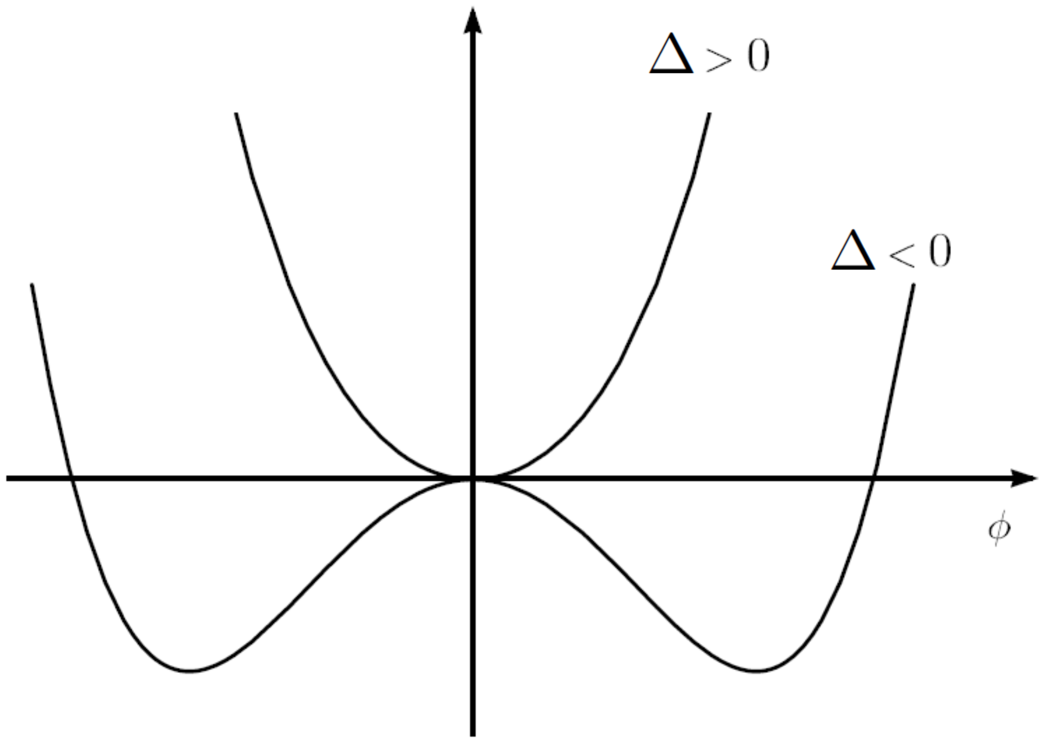
Кроссовер из CCD в QCD



Критическая динамика

ϕ^4 **-модель** $U \approx \Delta(g)\phi^2 + v\phi^4$

$$\Delta(g) = (g - g_c)$$



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$$G^K(\omega) = \langle \phi^{cl}(\omega)\phi^{cl}(-\omega)\rangle = \langle \phi^{cl}\phi^{cl}\rangle_\omega = \frac{2\gamma\omega\coth(\omega/T)}{\gamma^2\omega^2 + \varepsilon^2}$$

a

$$G^A(\omega) = \langle \phi^{q}(\omega)\phi^{cl}(-\omega)\rangle = \langle \phi^{q}\phi^{cl}\rangle_\omega = \frac{1}{i\gamma\omega + \varepsilon}$$

b

$$G^R(\omega) = \langle \phi^{cl}(\omega)\phi^{q}(-\omega)\rangle = \langle \phi^{cl}\phi^{q}\rangle_\omega = \frac{1}{-i\gamma\omega + \varepsilon}$$

c

Критическая динамика
(классическо-квантовый кроссовер)

ϕ^4 **-модель** $\quad U \approx \Delta(g)\phi^2 + v\phi^4$

$$G^K(\omega) = \frac{2\gamma\omega\coth(\omega/T)}{(\omega^2+\varepsilon_k)^2+\gamma^2\omega^2} \approx \frac{2\gamma T\,|\omega/T|^{\Lambda(T)}}{(\omega^2+\varepsilon_k)^2+\gamma^2\omega^2}$$

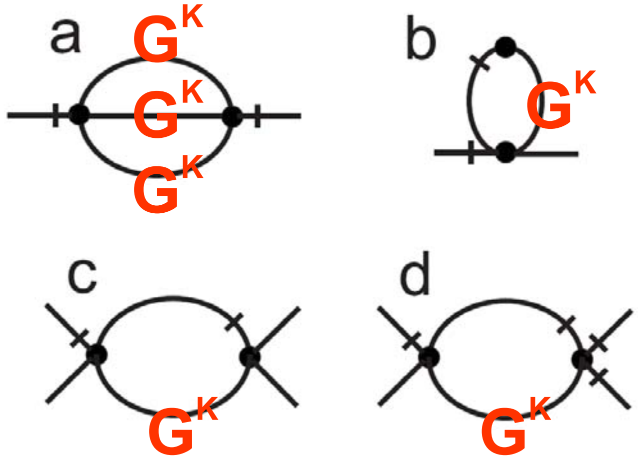
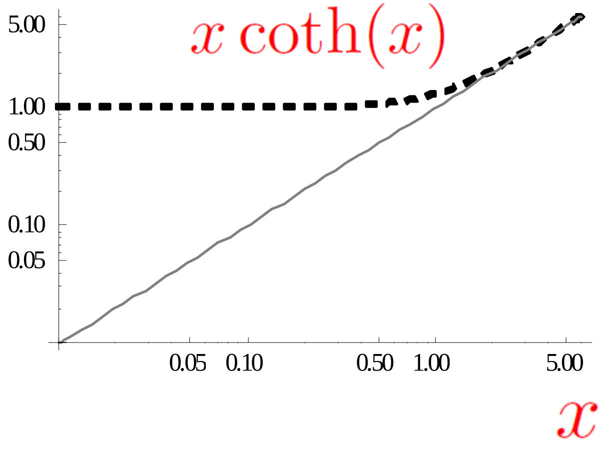


FIG. 3: The graph representation of the contributions to the renormalization of the theory's vertexes.

$$\sim \int \frac{|\omega|^{\Lambda}}{(k^2+i\omega)^3}\mathbf{d}\omega\mathbf{d}^d k$$

$$\Downarrow$$

$$\boxed{d \rightarrow d' = d + 2\Lambda}$$

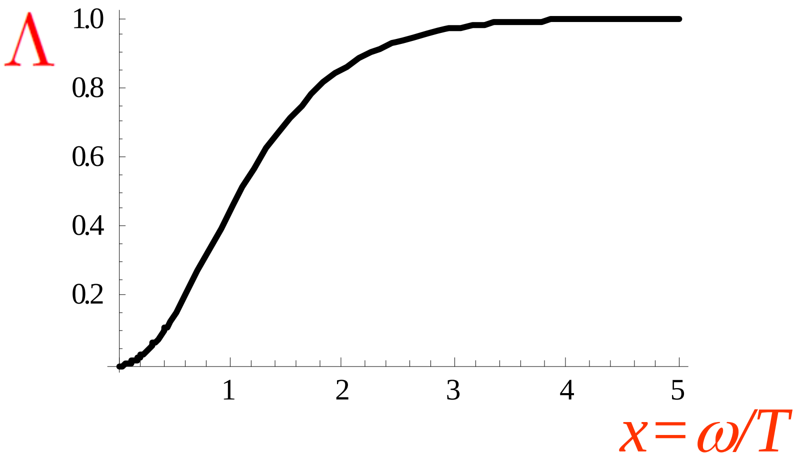


$$G^K(\omega) \approx \frac{2\gamma T \, |\omega/T|^{\Lambda(T)}}{(\omega^2+\varepsilon_k)^2+\gamma^2\omega^2}$$

$$\Lambda(\omega_0/T) \;=\; \partial \ln[x\coth(x)]/\partial \ln x\,|_{x=\omega_0/T} \quad (0 \leqslant \Lambda \leqslant 1)$$

$$\Downarrow$$

$$\Lambda(\omega_0/T) = (\coth(\omega_0/T) - (\omega_0/T)\,\mathrm{csch}^2(\omega_0/T))\tanh(\omega_0/T)$$



$$\boxed{d \longrightarrow d' = d + 2\Lambda}$$

Критические показатели:

$$\chi \sim |\Delta|^{-\gamma}$$ - susceptibility

$$r_c \sim |\Delta|^{-\nu}$$ - correlation radius

$$G(r) \sim r^{-d+2-\eta}$$ - Green function

$$C_v \sim |\Delta|^{-\alpha}$$ - heat capacity

$$\langle\phi\rangle \sim |\Delta|^{\beta}$$ - average field

Критические показатели:

$$\alpha' \approx \frac{4-n}{2(n+8)}\varepsilon', \qquad n = 1$$

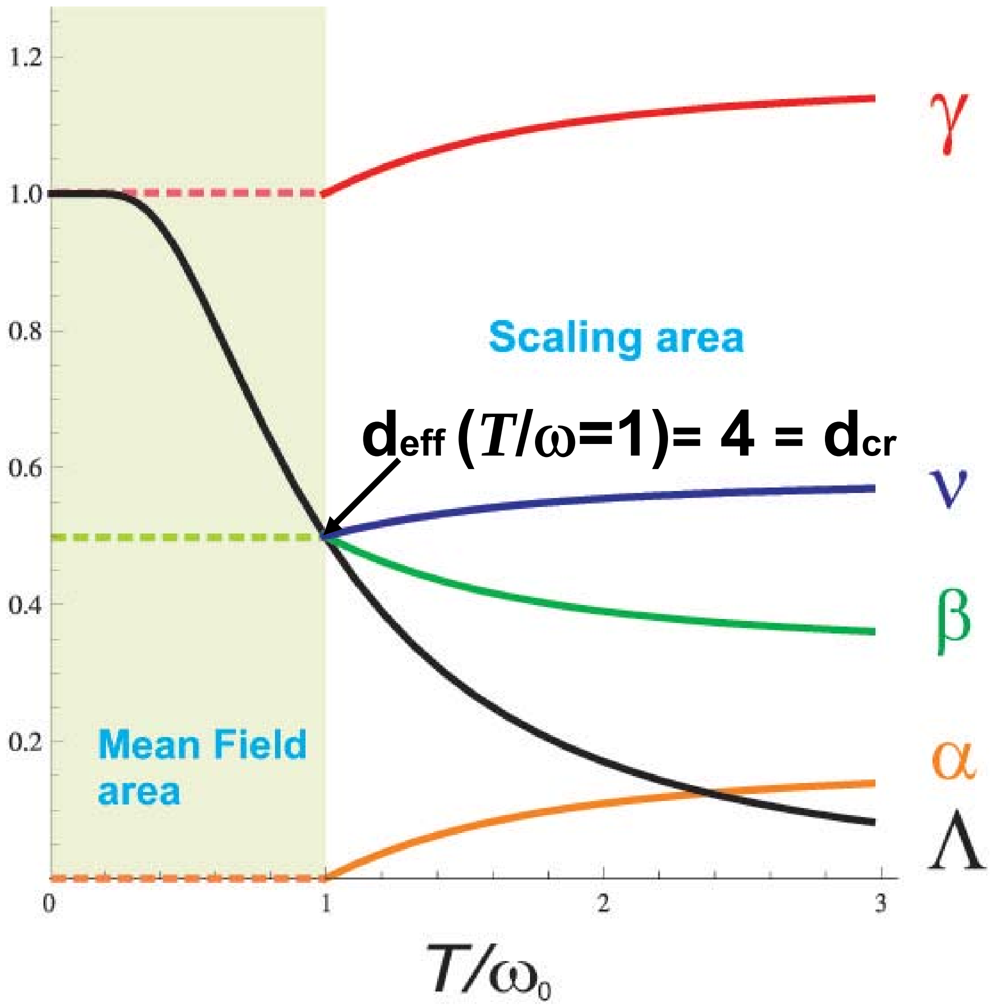
$$\nu' \approx \frac{1}{2} + \frac{n+2}{4(n+8)}\varepsilon',$$

$$\beta' \approx \frac{1}{2} - \frac{3}{2(n+8)}\varepsilon'$$

$$\gamma' \approx 1 + \frac{n+2}{2(n+8)}\varepsilon'$$

$$\varepsilon' = \varepsilon - \Lambda \qquad \varepsilon = (4-d)/2$$

Критические показатели (3D, Bose, n=1):



Выводы

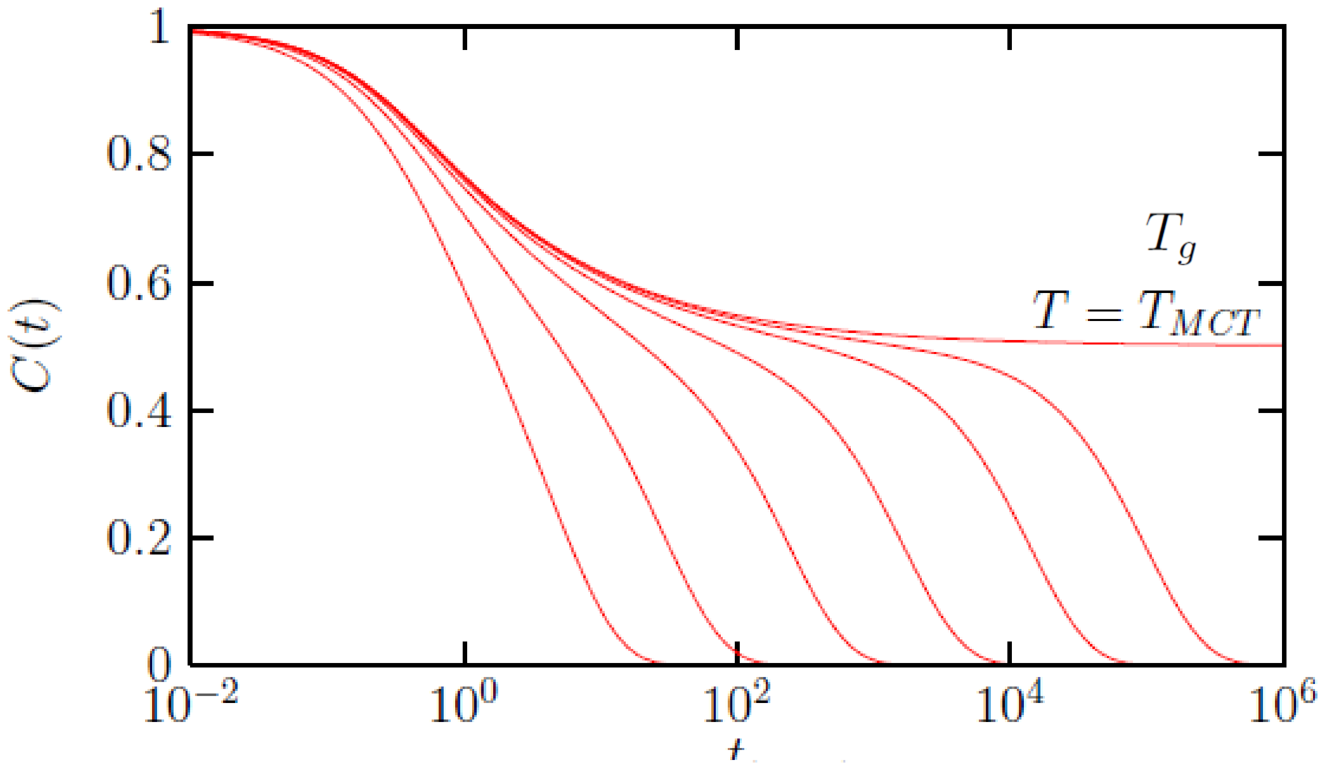
Вблизи квантовой критической точки существует три различных критических режима: классический диссипативный, квантовый диссипативный и квантовый недиссипативный.

Вблизи квантовой критической точки при приближении T к T=0 значения критических индексов непрерывно приближаются к крит. индексам, соответствующим размерности d+z

Теоретическое описание критической динамики перехода жидкость-стекло

Наличие плато на временной зависимости корреляционной функции

$$C(t) = \frac{1}{N}\sum_{i=1}^{N} < S_i(0)S_i(t) > \qquad C(t) = \frac{1}{V}\int d\mathbf{r}\langle\delta\rho(\mathbf{r},t)\delta\rho(\mathbf{r},0)\rangle$$



[Barrat *et al.*, 2003] *Slow relaxations and nonequilibrium dynamics in condensed matter*, Eds: J.-L. Barrat, J. Dalibard, M. Feigelman, J. Kurchan (Springer, Berlin, 2003).

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Conditions for glass transition in a system :

1. **The system is in a fluctuation region close to a supposed second order phase transition**

potential energy - $$U(\mathbf{Q}) = \frac{1}{2}\mu^2\mathbf{Q}^2 + \frac{1}{4}v\mathbf{Q}^4,$$

order parameter

$$\hat{\mu}^2 = \alpha K_B(T - T_c)$$

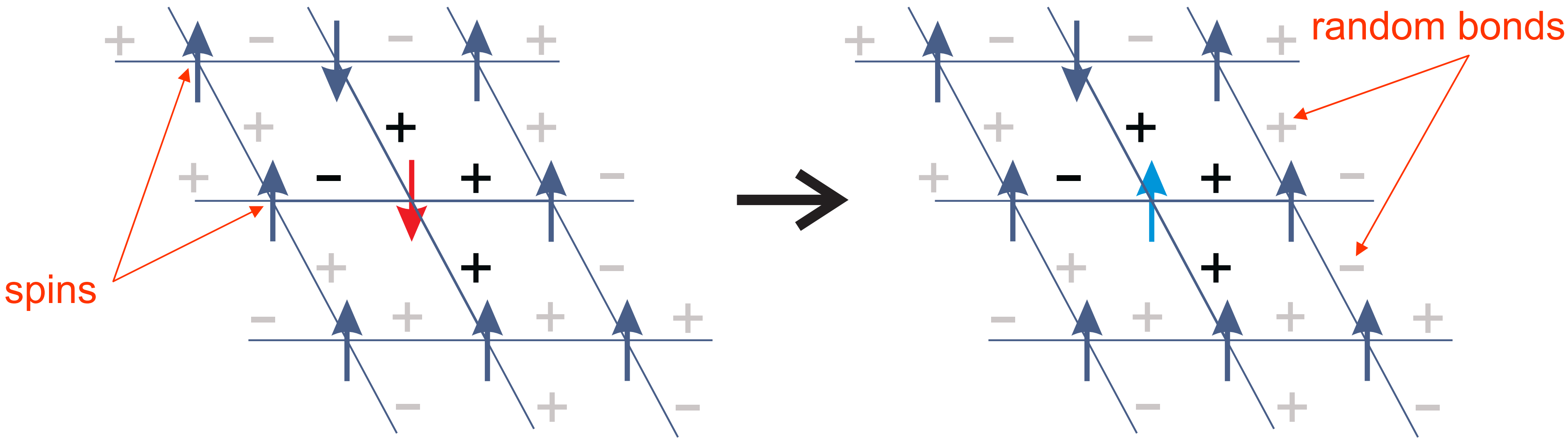
temperature of the system

temperature of phase transition

2. **The system is frustrated**

Frustration in spin glass

FIG. 1: The toy model of spin glass, described by the Edvards–Anderson Hamiltonian, $H = \sum J_{ij} S_i S_j$, where $S_i = \pm 1$ is a frustrated spin on the lattice point i, and $J_{ij} = \pm 1$. This system is invariant under the local transformation $J_{ij} \to -J_{ij}$ for all j adjacent to i.

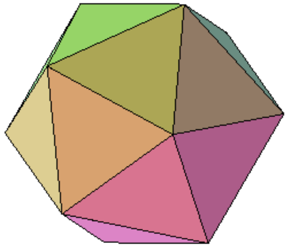


Local invariance **(in some points)**

Toulouse, G., 1977, Commun. Phys. 2, 115.

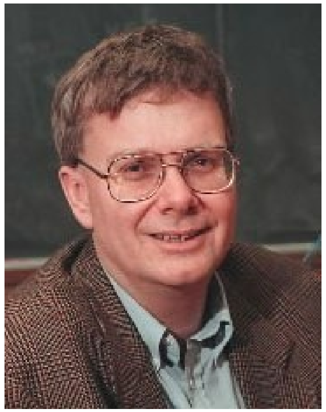
Frustration in amorphous matter

Geometrical frustration



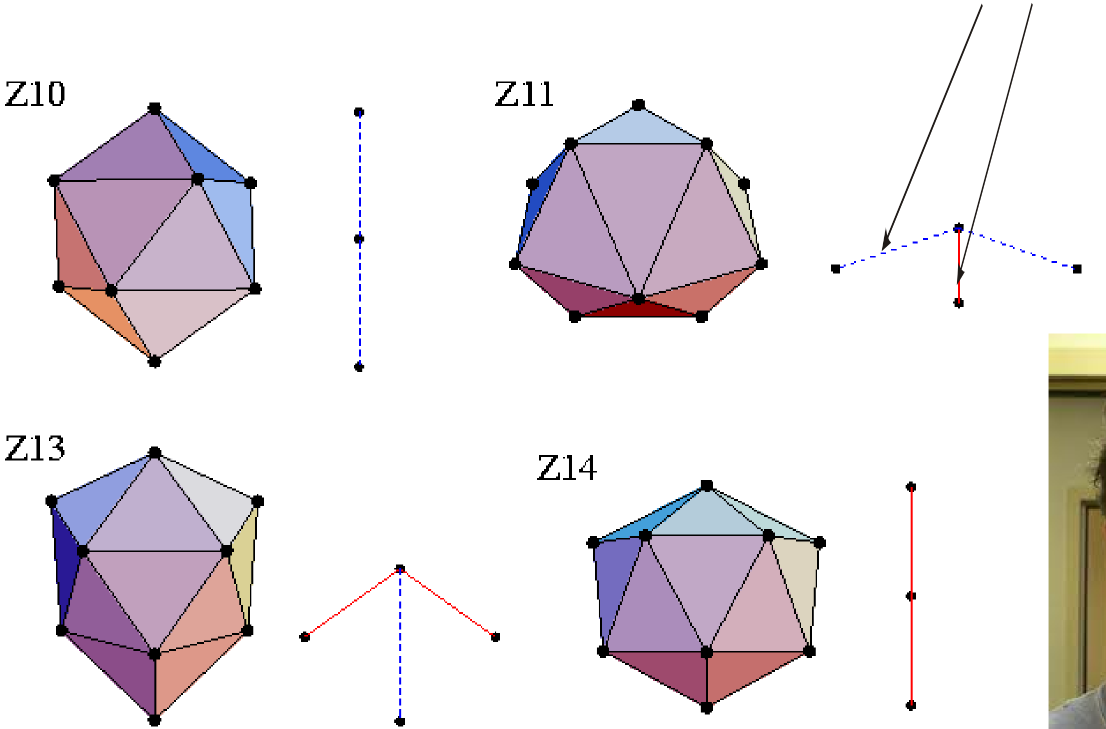
- icosahedra

Local invariance



David R. Nelson

disclinations

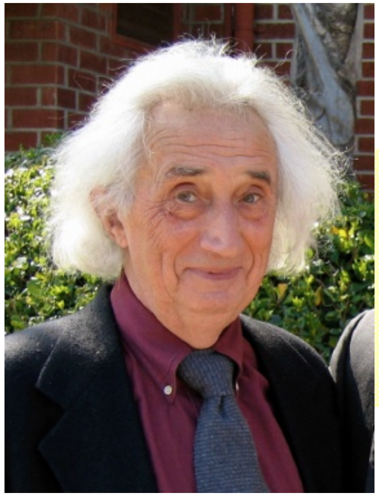


Gilles Tarjus

Z - coordination number

Gauge model of Glasses

Gauge model of Glass



I. E. Dzyaloshinskii

LE JOURNAL DE PHYSIQUE TOME 39, JUIN 1978, PAGE 693

ON THE CONCEPT OF LOCAL INVARIANCE IN THE THEORY OF SPIN GLASSES

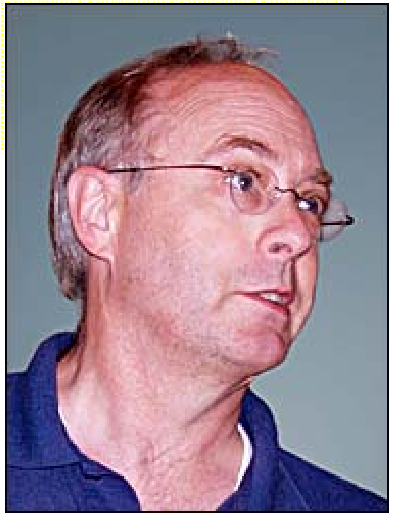
I. E. DZYALOSHINSKII and G. E. VOLOVIK

G. E. Volovik

PHYSICAL REVIEW B VOLUME 18, NUMBER 9 1 NOVEMBER 1978

Gauge models for spin-glasses

J. A. Hertz



J. A. Hertz

N. Rivier

J. Physique **43** (1982) 293-306

Line defects and tunnelling modes in glasses

N. Rivier and D. M. Duffy

Revista Brasileira de Física, Vd. 15, nº 4, 1985

Theory of Glass*

N. RIVIER

$$L = \frac{1}{2}(\partial_i \vec{s})(\partial_i \vec{s})$$

$$\vec{s}(\vec{r}) \longrightarrow \hat{R}\vec{s}(\vec{r})$$

The IDEA is to write a Lagrangian which is invariant relative to local rotation in r

$$\vec{s}(\vec{r}) \longrightarrow \hat{R}(\vec{r})\vec{s}(\vec{r})$$

$$\partial_i \vec{s}(\vec{r}) \longrightarrow \hat{R}(\vec{r})\partial_i \vec{s}(\vec{r}) + \left(\partial_i \hat{R}(\vec{r})\right)\vec{s}(\vec{r})$$

Local (gauge) invariance of action

One can introduce a new operator, which is invariant relatively of the local rotations:

$$D_i\vec{s}(\vec{r}) \to \hat{R}(\vec{r})D_i\vec{s}(\vec{r})$$

which has the form of:

$$D_i = \partial_i + \hat{A}_i(\vec{r})$$

and is determined by the expression:

$$\left(\partial_i\hat{R}(\vec{r})\right)\vec{s} = -[\hat{A}\hat{R}]\vec{s}$$

Local (gauge) invariant action

Then, in continuous presentation, the derivative should be replaced by the covariant derivative with some gauge field. As a result the action has the form of

sources of the gauge field

$$\mathcal{S} = \beta \int \left[\frac{1}{2} (\vec{D}\mathbf{Q})^2 - U(\mathbf{Q}) + \frac{1}{4} \mathbf{F}^2 + \mathbf{J}\mathbf{A} \right] d\mathbf{r},$$

where

covariant derivative

gauge field

$$U(\mathbf{Q}) = \frac{1}{2} \mu^2 \mathbf{Q}^2 + \frac{1}{4} v \mathbf{Q}^4,$$

$$D_i Q_{lk} = \partial_i Q_{lk} + g \varepsilon_{iab} A_{la} Q_{kb},$$

$$F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + g \varepsilon_{abc} A_{b\mu} A_{c\nu}.$$

Averaging over fast modes (FMA)

$$\mathcal{S} = \beta \int \left[\frac{1}{2}(\vec{D}\mathbf{Q})^2 - U(\mathbf{Q}) + \frac{1}{4}\mathbf{F}^2 + \mathbf{J}\mathbf{A} \right] d\mathbf{r}$$

1) **J** – are sources of the gauge field (vortex, disclinations) appearing because of the **frustration.** At **T>Tc** they are in thermal equilibrium, therefore one can average over **J**

$$\boxed{\langle \mathbf{J}\mathbf{J} \rangle_k = I_0,}$$

density of frustrations

2) Close to the **Tc** the order parameter is represented as a sum of the **slow** and **fast** parts.

$$\mathbf{Q} = \mathbf{\Phi} + \mathbf{\Psi}$$

slow part fast part

We should to carry out the averaging over fast part

$$\langle \mathbf{\Psi}^2 \rangle \approx \hat{\mu}^2 / 2v$$

The result of FMA

$$\mathcal{S}' = \beta \int \left[\frac{1}{2}(\vec{\partial}\mathbf{\Phi})^2 + \frac{g^2}{2}\mathbf{\Phi}^2\mathbf{A}^2 + \frac{1}{2}M^2\mathbf{A}^2 + \frac{1}{2}\hat{\mu}^2\mathbf{\Phi}^2 + \frac{1}{4}v\mathbf{\Phi}^4 + \frac{1}{4}\mathbf{F}^2 \right] d\mathbf{r}$$

$$\hat{\mu}^2 = \alpha K_B (T - T_c)$$

$$M^2 = \hat{\mu}^2 g^2 / 2v - I_0$$

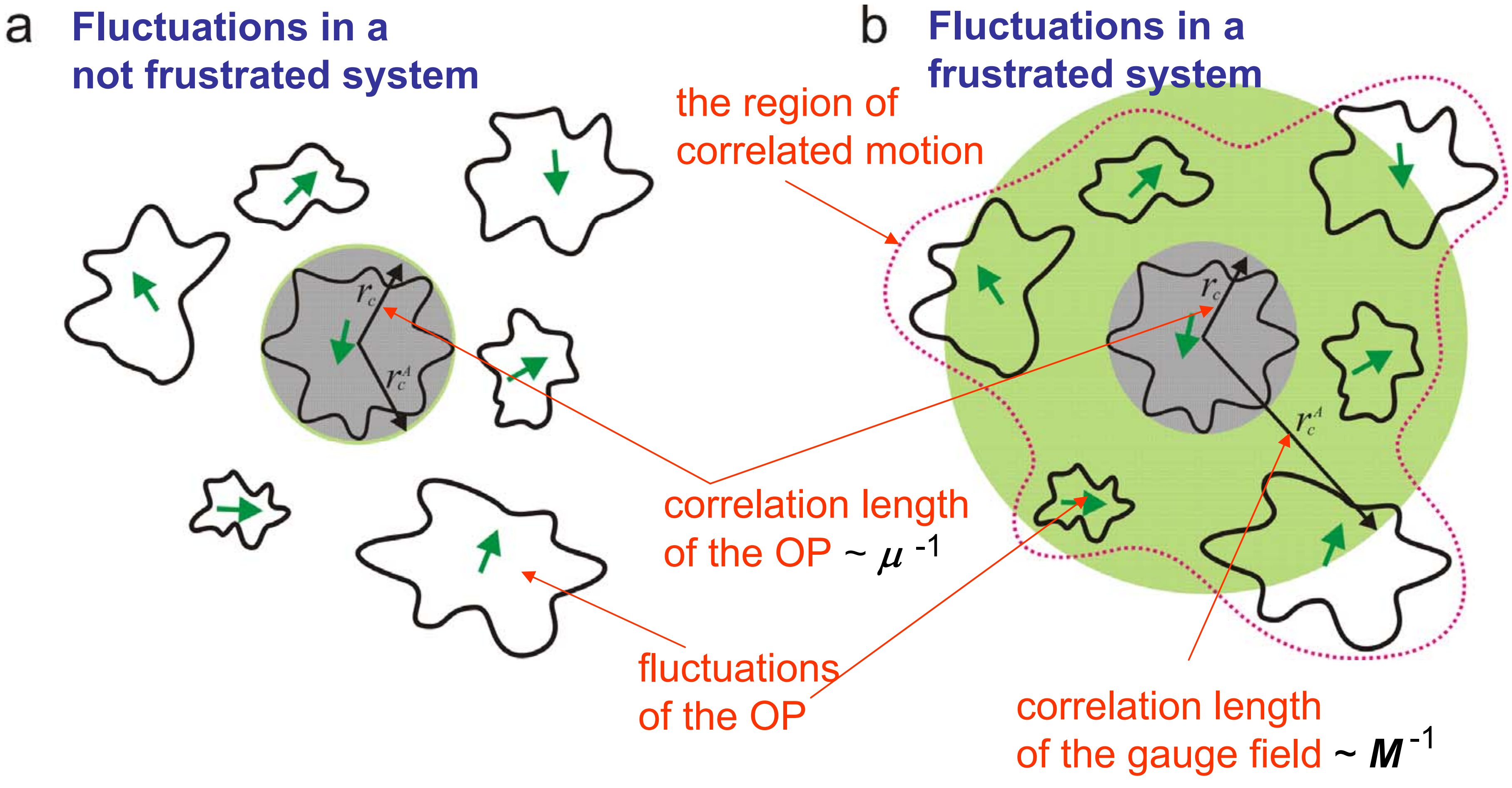
As a result the transition happens at $M^2 = 0$

and the temperature of this transition shifts from T_c, to T_g > T_c.

$$T_g = T_c + 2 I_0 v / \alpha K_B g^2$$

The physical meaning of the model

$$\mathcal{S}' = \beta \int \left[\frac{1}{2}(\vec{\partial}\mathbf{\Phi})^2 + \frac{g^2}{2}\mathbf{\Phi}^2\mathbf{A}^2 + \frac{1}{2}M^2\mathbf{A}^2 + \frac{1}{2}\hat{\mu}^2\mathbf{\Phi}^2 + \frac{1}{4}v\mathbf{\Phi}^4 + \frac{1}{4}\mathbf{F}^2 \right] d\mathbf{r}$$



$T \approx T_c$ **$T \approx T_g$**

$$\vec{\phi} = \{\bar{\phi}, \phi\}, \quad \vec{A}^a_\mu = \{\bar{A}^a_\mu, A^a_\mu\}$$

$$Z = \int D\vec{\phi} D\vec{A}^a_\mu \exp\left\{\frac{i}{2}\int_{t,t'} \vec{\phi}(t)\hat{G}^{-1}(t, t')\vec{\phi}(t') + \right.$$
$$\frac{i}{2}\int_{t,t'} \vec{A}^a_\mu(t)\hat{\Delta}^{-1}_{\mu\nu}(t, t')\vec{A}^a_\nu(t') - ig\varepsilon^{abc}\int_t(\partial_\mu \bar{A}^a_\nu)A^b_\mu A^c_\nu -$$
$$ig\varepsilon^{abc}\int_t(\partial_\mu A^a_\nu)\bar{A}^b_\mu A^c_\nu - ig\varepsilon^{abc}\int_t(\partial_\mu A^a_\nu)A^b_\mu \bar{A}^c_\nu -$$
$$ig^2\varepsilon^{abc}\varepsilon^{aij}\int_t \bar{A}^b_\mu A^c_\nu A^i_\mu A^j_\nu -$$
$$\left. ig^2\int_t \bar{A}^a_\mu A^a_\mu \phi^2 - ig^2\int_t(A^a_\mu)^2\bar{\phi}\phi - iv4\int_t \bar{\phi}\phi^3\right\},$$

$$M^2 = 0$$
$$\mu^2 = 4I_0 v/g^2$$

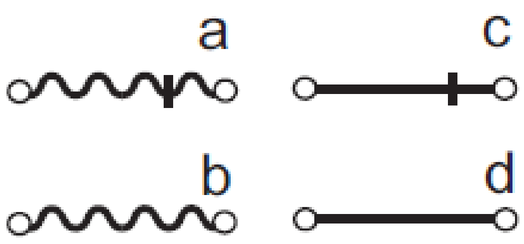
Функциональные методы (метод динамического производящего функционала, метод Келдыша)

$$\vec{\phi} = \{\bar{\phi}, \phi\}, \; \vec{A}^a_\mu = \{\bar{A}^a_\mu, A^a_\mu\}$$

$$\hat{G} = \begin{pmatrix} G^K & G^A \\ G^R & 0 \end{pmatrix}, \quad \hat{\Delta}_{\mu\nu} = \begin{pmatrix} \Delta^K_{\mu\nu} & \Delta^A_{\mu\nu} \\ \Delta^R_{\mu\nu} & 0 \end{pmatrix}.$$

$$i\Delta^{R(A)}_{\mu\nu}(k,\,\omega) = \frac{-i\delta_{\mu\nu}}{k^2 + M^2 \pm i\Gamma_A\omega}, \qquad iG^{R(A)}(k,\,\omega) = \frac{i}{k^2 + \mu^2 \pm i\Gamma_\phi\omega},$$

$$i\Delta^K_{\mu\nu}(k,\,\omega) = \frac{-i2\Gamma_A\delta_{\mu\nu}}{(k^2 + M^2)^2 + \Gamma_A^2\omega^2}, \qquad iG^K(k,\,\omega) = \frac{i2\Gamma_\phi}{(k^2 + \mu^2)^2 + \Gamma_\phi^2\omega^2},$$



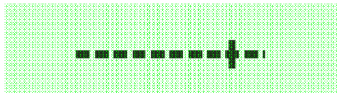
For analysis of the system's critical properties close to T_g one can use the functional methods of non-equilibrium dynamics

(Keldysh technique)

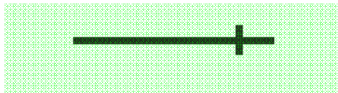
$$\vec{A}^a_\mu = \{\bar{A}^a_\mu,\, A^a_\mu\} \qquad\qquad \vec{\Phi} = \{\bar{\Phi},\, \Phi\}$$

$$\hat{\Delta}_{\mu\nu} = \begin{pmatrix} \Delta^K_{\mu\nu} & \Delta^A_{\mu\nu} \\ \Delta^R_{\mu\nu} & 0 \end{pmatrix} \qquad\qquad \hat{G} = \begin{pmatrix} G^K_0 & G^A_0 \\ G^R_0 & 0 \end{pmatrix}$$

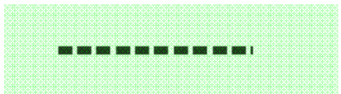
$$\Delta^A_{\mu\nu} = \langle \bar{\mathbf{A}} \mathbf{A} \rangle_t = \theta(t) \frac{e^{-t\varepsilon_k(M)/\Gamma_A(T)}}{\Gamma_A(T)}$$



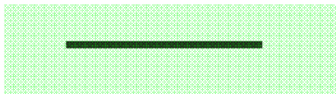
$$G^A_0 = \langle \bar{\Phi} \Phi \rangle_t = \theta(t) \frac{e^{-t\varepsilon_k(\mu)/\Gamma_\Phi(T)}}{\Gamma_\Phi(T)}$$



$$\Delta^K_{\mu\nu} = \langle \mathbf{A} \mathbf{A} \rangle_t = \frac{e^{-|t|\varepsilon_k(M)/\Gamma_A(T)}}{\varepsilon_k(M)}$$

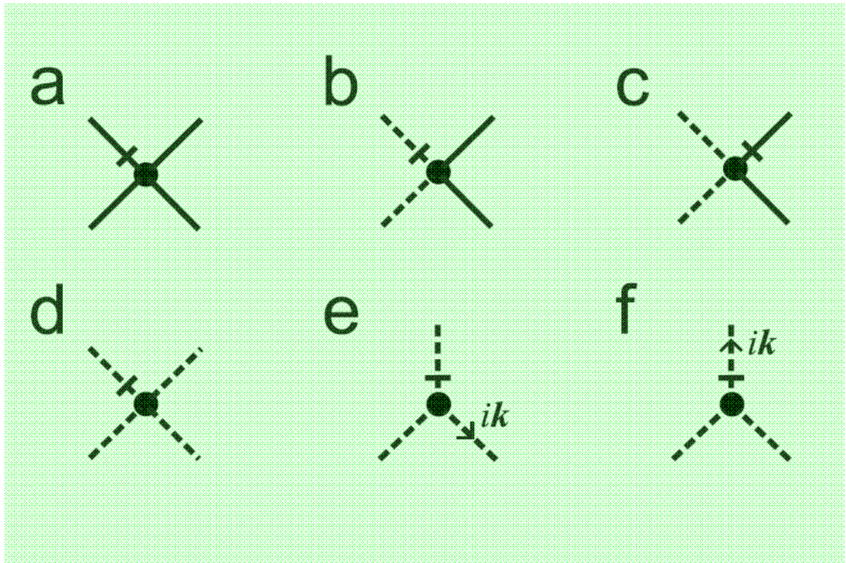


$$G^K_0 = \langle \Phi \Phi \rangle_t = \frac{e^{-|t|\varepsilon_k(\mu)/\Gamma_\Phi(T)}}{\varepsilon_k(\mu)}$$

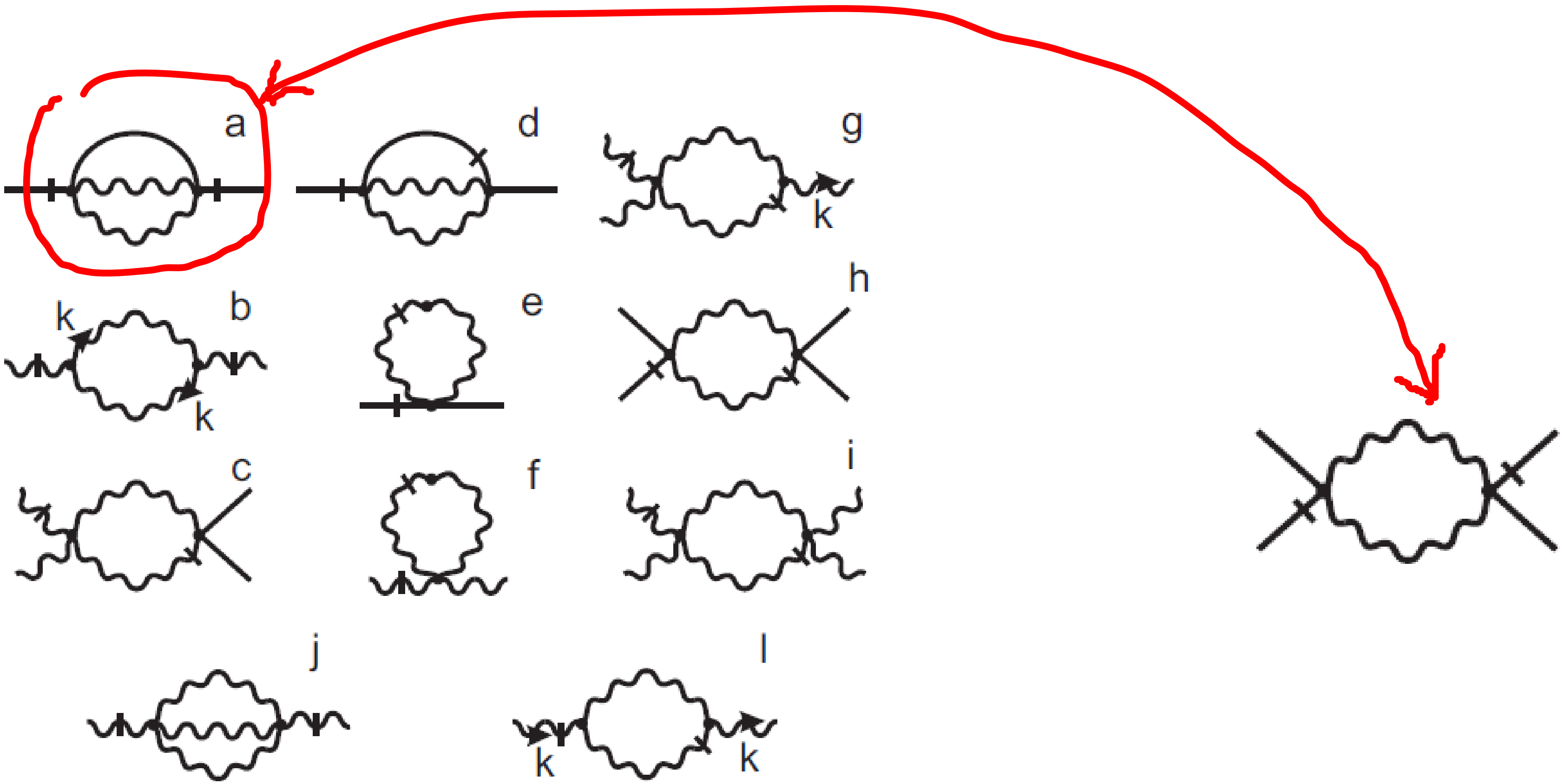


$$\boxed{\varepsilon_k(x) = k^2 + x^2}$$

вершины



Перенормировка



$$\Sigma \approx \Gamma_\phi \frac{g^4 \ln(1/\Lambda)}{\pi^2}\left(1 - e^{-4I_0 v|t_0|/\Gamma_\phi g^2}\right)$$

Λ - параметр регуляризации, t_0 - время наблюдения

$$t_o \gg \Gamma_\phi g^2/4I_0 v$$

$$\frac{\partial \ln(\Gamma_\phi)}{\partial \xi} = g^4/\pi^2,$$

$$\frac{\partial \ln(\Gamma_A)}{\partial \xi} = 3g^4/\pi^2 + g^2/2\pi^2,$$

$$\frac{\partial \ln(M^2)}{\partial \xi} = 2 + 3g^2/2\pi^2,$$

$$\frac{\partial \ln(\mu^2)}{\partial \xi} = 2 + \frac{M^2 g^2}{2\mu^2\pi^2} \approx 2,$$

$$\frac{\partial \ln(g^2)}{\partial \xi} = \varepsilon - g^2/\pi^2,$$

$$\frac{\partial \ln v}{\partial \xi} = \varepsilon - g^4/2v\pi^2,$$

$$t_o \ll \Gamma_\phi g^2/4I_0 v$$

$$\frac{\partial \ln(\Gamma_\phi)}{\partial \xi} = 0.$$

$$\frac{\partial \ln(\Gamma_A)}{\partial \xi} = 3g^4/\pi^2 + g^2/2\pi^2,$$

$$\frac{\partial \ln(M^2)}{\partial \xi} = 2 + 3g^2/2\pi^2,$$

$$\frac{\partial \ln(\mu^2)}{\partial \xi} = 2 + \frac{M^2 g^2}{2\mu^2\pi^2} \approx 2,$$

$$\frac{\partial \ln(g^2)}{\partial \xi} = \varepsilon - g^2/\pi^2,$$

$$\frac{\partial \ln v}{\partial \xi} = \varepsilon - g^4/2v\pi^2,$$

ренормгруппа

$$\xi = \ln(1/\Lambda)$$

$$\frac{\partial \ln(\Gamma_\phi)}{\partial \xi} = g^4 S(\xi)/\pi^2,$$

$$S(\xi) = 1 - \Lambda^z = 1 - \exp(-z\xi),$$

$$z \approx 2$$

В окрестности критической точки T_g

$$M^2 = \alpha k_B(T - T_g) \approx e^{2\xi} \qquad g^2 = \pi^2\varepsilon \quad v = g^2/2$$

$$\boxed{\tau_{rel} = \Gamma_\phi \propto \exp\left(\frac{g^4 T_g}{2\alpha\pi^2(T - T_g)}\right).}$$

Vogel–Tammann–Fulcher (VTF)

M.G. Vasin, J. Stat. Mech., P05009 (2011);
M.G. Vasin, N.M. Shchelkachev, and V.M. Vinokur, Theoretical and Mathematical Physics, 163(1): 537–548 (2010);

The ergodicity breaking is taken into account in process of the renormalization group analysis of this model in terms of critical dynamics.

Results are presented in:

1. Vasin M.G., Physica A: Statistical Mechanics and its Applications, v.431, 18–28 (2015)

1. Vasin M.G., Theoretical and Mathematical Physics, 174(3): 406–420 (2013);

2. Vasin M.G., Tareyeva E.E., Shchelkacheva T.I., and Shchelkatchev N.M., Theoretical and Mathematical Physics, 174(2): 197–208 (2013);

3. Vasin M.G., Journal of Physics: Conference Series 394 (2012) 012010;

4. Vasin M.G. J. Stat. Mech. P05009 (2011);

5. Vasin M.G., Chtchelkatchev N.M., Vinokur V.M. Theoretical and Mathematical Physics, V.163, № 1, p.163-176 (2010);

Some results of the theory

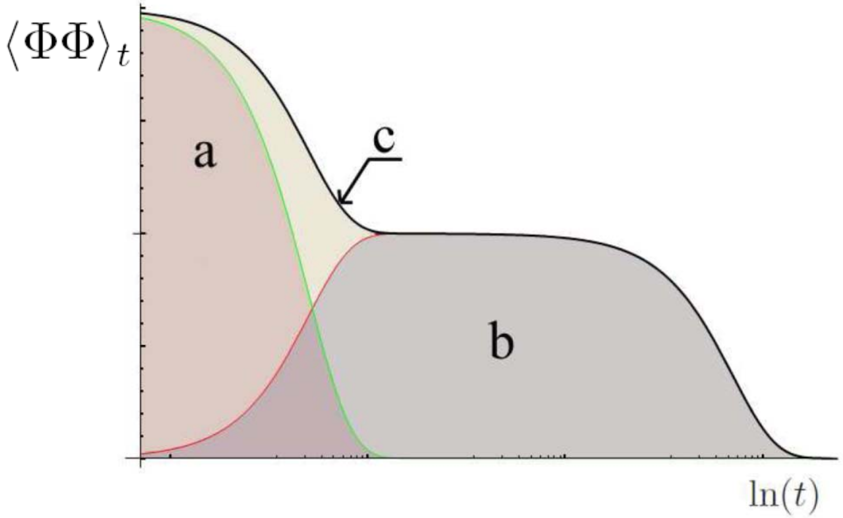
1. Vogel-Fulcher-Tamman law for temperature dependence of relaxation time

$$\tau_{rel} = \Gamma_\phi \propto \exp\left(\frac{2vg^2T_g}{\alpha\pi^2(T-T_g)}\right)$$

2. The observed glass transition temperature T_g^* depends on the cooling rate v.

$$T_g^*(\mathrm{v}) = T_g + \frac{T_gC}{2\ln x - \ln(\mathrm{v}\tau_o/T_g)}$$

3. The plateau on the time dependence of the correlation function



Some results of the theory

4. Linear susceptibility does not diverge in Tg

$$\chi_L = \partial\langle\Phi\rangle/\partial h \propto \mu^{-2} = g^2/3I_0 v$$

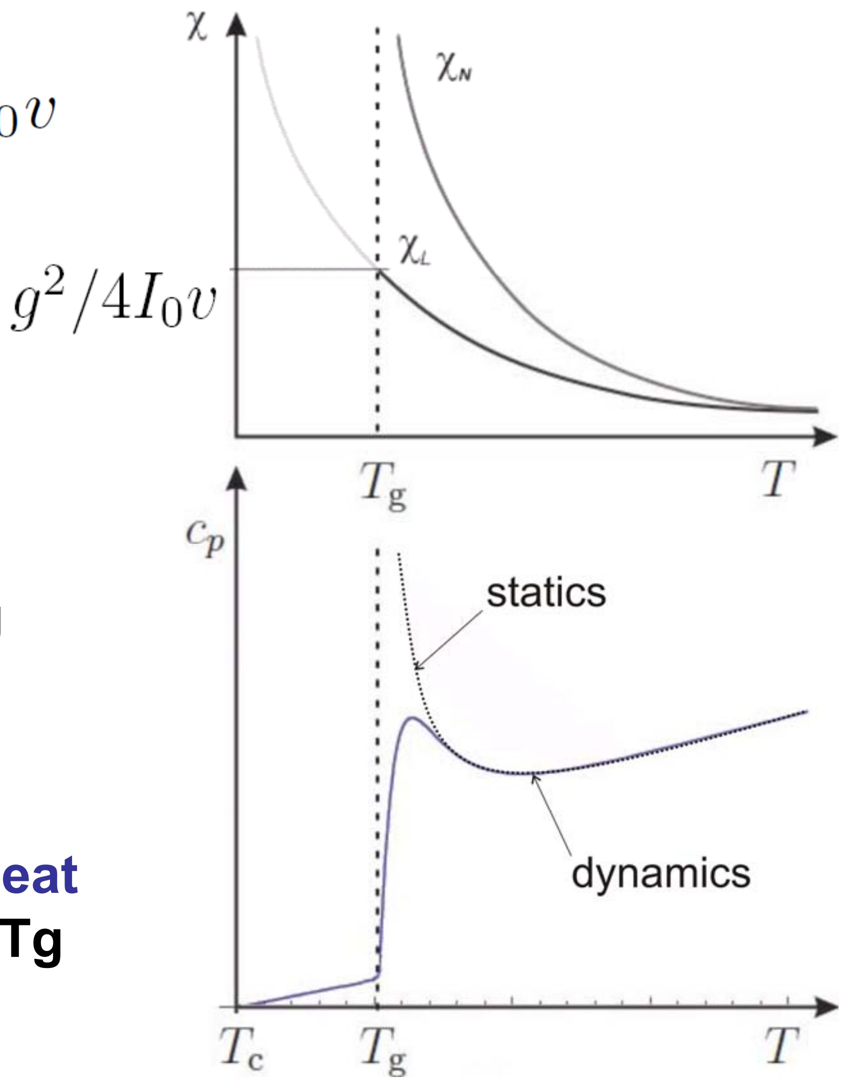
5. Non-linear susceptibility diverges in Tg

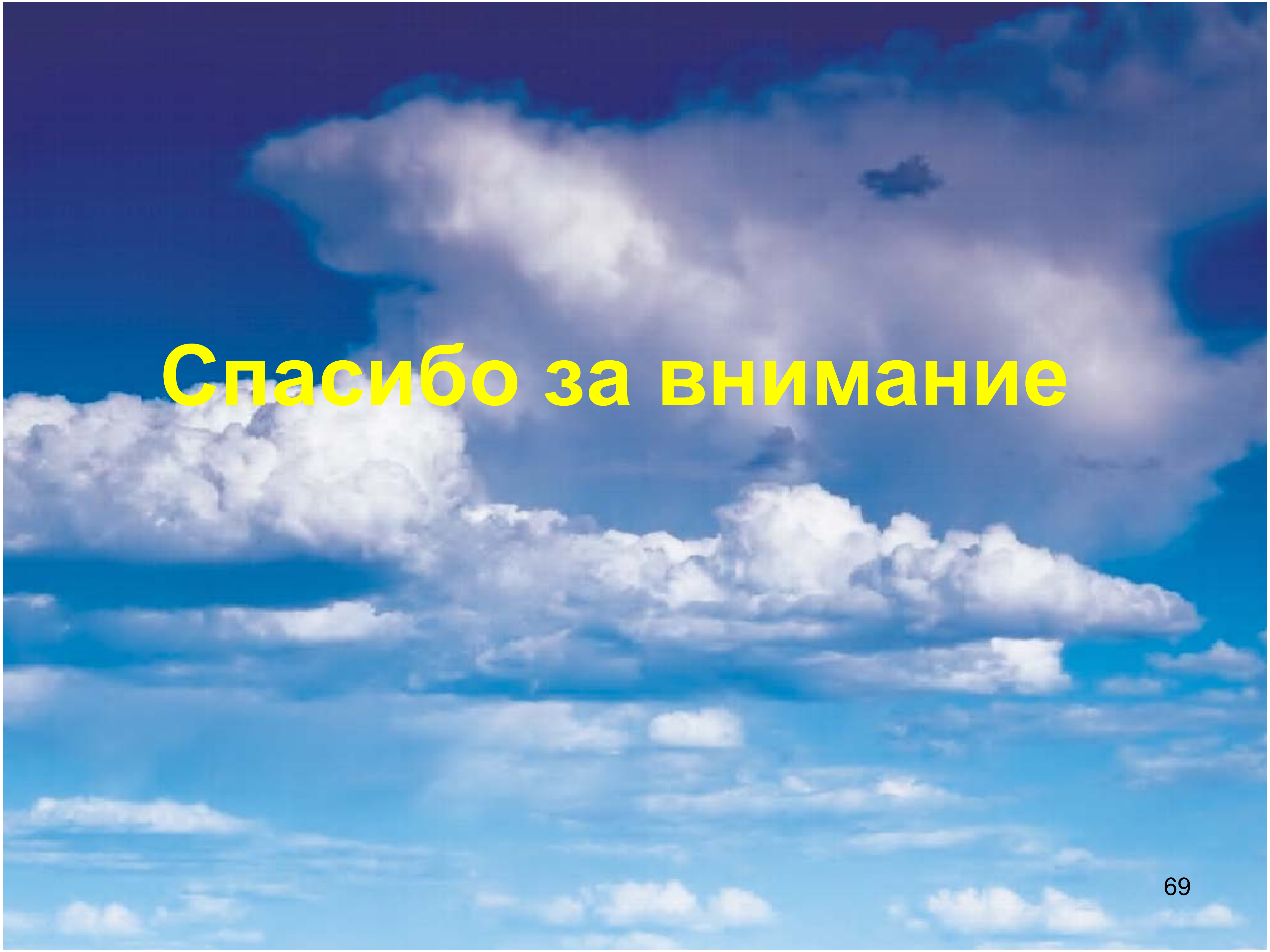
$$\chi_N = \partial^3\langle\Phi\rangle/\partial h^3 = \langle\Phi^4\rangle$$

6. The size of ordered regions at Tg

$$r_{cor} \sim \sqrt{g^2/4I_0 v}$$

7. The temperature function of the heat capacity can have a maximum near Tg




Спасибо за внимание